

Glacial Isostatic Adjustment Observed with VLBI and SLR

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Abstract

In global geodetic solutions vertical rates of site motion are usually estimated relative to the geocenter (center of figure) of the solid earth. The velocity of the geocenter is estimated assuming that the plates are rigid, that the velocities of the plates equal those in NUVEL-1A [DeMets et al. 1990, 1994], and that the uplift, subsidence, and intraplate deformation due to glacial isostatic adjustment is negligible. In this article we estimate the velocity of the geocenter assuming only that the uplift, subsidence, and deformation of the plate interiors equals that predicted by the glacial isostatic adjustment model of Peltier [1994] or that of Peltier [1996]. Using geodetic data from very long baseline interferometry (VLBI) and satellite laser ranging (SLR) taken over twenty years, we estimate vertical rates of site motion relative to the geocenter assuming that the velocity of the geocenter relative to the center of mass is negligible when averaged over decades. The VLBI and SLR data observe the isostatic adjustment of the solid earth in response to unloading of the late Pleistocene ice sheets. Onsala (Sweden) is rising at 3 mm/yr in response to unloading of the Fennoscandian ice sheet and Algonquin Park (Ontario) is rising at 2 mm/yr in response to unloading of the Laurentide ice sheet. The data tightly limit the gradient in uplift (to subsidence) rate going away from the center of the Laurentide ice sheet in eastern North America. The forebulge of the Laurentide ice sheet is, along the northeast U.S. coast, presently subsiding very slowly. The margins of the ice sheets are moving away from their centers very slowly if at all. This observation disagrees with the model of Peltier [1996], which predicts the margins of the ice sheets to be moving away from the centers at a few mm/yr.

Introduction

Models of Glacial Isostatic Adjustment

The isostatic response of the solid earth in response to unloading of the late Pleistocene ice sheets is evident in elevated beach terraces in northern Europe and Canada. Radiocarbon dating of these terraces yield relative sea level histories [e.g., Pirazolli 1991], providing the basis for viscoelastic models of glacial isostatic adjustment accounting for both the transformation of ice sheets into ocean water and the gravitational

effects of changes in the solid earth on the sea surface [Peltier 1994, 1996] (Figures 1 and 2). These postglacial rebound models involve two unknown parameters, the mass of the ice sheets as a function of time and the viscosity of the mantle as a function of depth. Using only relative sea level histories to constrain the model leads to highly correlated estimates of deglaciation history and mantle viscosity. Errors in the knowledge of either unknown propagates into the inference of the other. Thus the wide range of values for mantle viscosity in the literature may be a simple consequence of errors in deglaciation history. Similarly, models of deglaciation history are sensitive to errors in the radial variation in mantle viscosity.

In this article we compare geodetic results with the postglacial rebound models of Peltier [1994] and Peltier [1996]. The model of Peltier [1994] is determined from relative sea level histories at 414 places, about half of which were beneath ice during the last glacial maximum 21 thousand years ago. The elastic structure of the laterally-invariant viscoelastic model is assumed to be that of the Preliminary Reference Earth Model [PREM, Dziewonski and Anderson, 1981]. The viscous structure is assumed to consist of three layers, an elastic lithosphere 120 km thick, an upper mantle and transition zone with viscosity 1×10^{21} Pa s, and a lower mantle with viscosity 2×10^{21} Pa s (Figure 3). The deglaciation history estimated assuming this earth rheology is ICE-4G.

The postglacial rebound model of Peltier [1996] is determined from a range of geophysical data that allow detailed variations in mantle viscosity as a function of depth to be estimated [see also Peltier and Jiang 1996]. The data consist of relative sea level histories, the relaxation spectrum for Fennoscandian rebound [McConnell 1968], the ongoing wander of the earth's (north) axis of rotation at the rate of $\sim 0.95^\circ/\text{Myr}$ along the 76°W meridian [Vicente and Yumi 1969], and the so-called nontidal acceleration of the rate of rotation [Stephenson and Morrison 1995]. The estimated earth rheology has an upper mantle less viscous and a lower mantle more viscous than in Peltier [1994] (Figure 3). The ice deglaciation history is identical in North America to that in Peltier [1994] but has a slightly thicker ice sheet in Fennoscandia during the last glacial maximum.

In this article we compare geodetic results to only the models of Peltier [1994, 1996] for a number of reasons. We know of no other full rebound model, with both deglaciation history and mantle viscosity profile, whose predictions are available. Peltier [1998] shows that the mantle viscosity profiles of Lambeck et al. [1990], Forte and Mitrovica

[1996], and Simons and Hager [1997] all overestimate the characteristic time describing the exponential decay in uplift evident in relative sea level histories along the southeast coast of Hudson Bay. Peltier [1998] furthermore maintains that the model of Mitrovica and Forte [1997] poorly fits the slow subsidence of the east U.S. coast evident in relative sea level histories. Our primary reason for comparing only to Peltier [1994] and Peltier [1996] is that in this article we seek to describe the spatial variation in glacial isostatic adjustment but do not attempt to rigorously limit mantle viscosity and deglaciation history by inverting the geodetic data.

In this article we use geodetic data from very long baseline interferometry (VLBI) and satellite laser ranging (SLR) to estimate velocities generated by glacial isostatic adjustment. Toward this aim we first describe how to place estimated vertical rates of site motion into the reference frame in which they should be compared with the rebound model predictions. We next compare reference frames determined using various definitions of the deep interior of the earth. We then describe the reduction of data for parameters. Finally we compare observations of site motions due to glacial isostatic adjustment with those predicted by the models of Peltier [1994] and Peltier [1996].

Methods and Data Reduction

Definition of the Reference Frame

We distinguish between two definitions of the earth center: (1) the geocenter and (2) the center of mass. The geocenter is the center of figure of the surface of the solid earth. The center of mass of the earth (solid earth, oceans, and atmosphere) is the mean point satellites orbit about. In this article we compare and contrast estimates of the vertical rates of site motion relative to the geocenter with those relative to the center of mass.

When interpreting a geodetic velocity solution it is important to consider how the translational component of the reference frame is defined [Heki 1996, Argus 1996]. Fixing a reference point (whether it be the geocenter, the center of mass, or the deep interior of the earth) is equivalent to fixing the translational component of the reference frame. Changing the velocity of the reference point relative to surface sites in one direction results in a rigid-body translation of all site velocity estimates relative to the reference point (Figure 4).

We seek to estimate vertical rates of site motion relative to the deep interior of the earth because this is the reference frame in which velocities predicted by the postglacial rebound models of Peltier [1994, 1996] and other scientists are described. The rates of uplift and subsidence of the plate interiors relative to the deep interior of the earth are probably minor aside from postglacial rebound. No other phenomenon is known to raise or lower major portions of the interior of the plates. We define the geocenter to be the point yielding no uplift or subsidence of the interiors of the plates after removing the effects of glacial isostatic adjustment (Figure 5, the geocenter as we define it differs from the center of figure). The velocity of the geocenter defined in this manner is equal to the velocity of the deep interior of the earth if glacial isostatic adjustment is the sole cause of uplift and subsidence of the plate interiors. Sites in the deformation belts are not used to constrain the velocity of the geocenter because the subsidence associated with rifting, the uplift associated with mountain building, and coseismic movements and postseismic transients associated with plate boundary processes would generate biases.

Doubts about how to specify the translational component of the reference frame for velocity are evident in the range of methods used. Ryan et al. [1993] and Ma et al. [1994] define the translational velocity of the reference frame for VLBI by fixing the vertical rates of Westford (Massachusetts), Richmond (Florida), and Kauai (Hawaii) to zero. Watkins et al. [1994] find that a translational velocity of 1.6 mm/yr places the VLBI solution of Ryan et al. [1993] into a reference frame defined by the center of mass as observed by SLR. Heki [1994] define the reference frame by minimizing differences between estimated site velocities and those predicted by no-net-rotation global plate motion model NNR-NUVEL1 [DeMets et al. 1990, Argus and Gordon 1991] and find a translational velocity differing from that of Ma et al. [1994] by 3.7 mm/yr. Ma and Ryan [1995] also define the translational velocity of the VLBI reference frame using no-net-rotation model NNR-NUVEL1A [DeMets et al. 1994]. Argus [1996] defines the reference frame by minimizing differences between vertical observables and postglacial rebound predictions [Peltier 1994] and finds a translational velocity differing from that of Ma and Ryan [1995] by 1.7 mm/yr.

Estimating vertical rates relative to the geocenter assuming that the uplift, subsidence, and deformation of the plate interiors is negligible after correcting for glacial

isostatic adjustment (model "Geoc."). Because the radio telescopes and laser stations very sparsely sample the surface of the solid earth, it is impossible to use them to estimate the position of the geocenter. If, however, we assume the plate interiors move tangent to the surface of the earth (that is, with no radial motion), we can estimate the velocity of the geocenter relative to the sphere (or ellipsoid) the plates rotate in. The isostatic response of the Earth to unloading of the late Pleistocene ice sheets creates radial (vertical) motions, violating the assumption that there are none.

In this article we estimate the velocity of the geocenter as follows. We assume that, aside from the deformation produced by postglacial rebound, the plates are rigid (no radial motion). We first subtract from all the observables the horizontal and vertical predictions of a particular rebound model. We next solve for the translational velocity minimizing the sum of squared differences between the modified observables and the plate model predictions. For a vertical observable the model prediction is the projection of the translational velocity onto the vertical at the site. For a horizontal observable the model prediction is the sum of a contribution from the translational velocity and a contribution from the angular velocity of the plate on which the site lies. The former is the projection of the translational velocity onto the horizontal at the site. The latter is the usual cross product between the angular velocity of the plate and the geocentric vector to the site.

The vertical data constrain the geocenter velocity because if the geocenter velocity is wrong, a plate on one side of the earth will appear to be rising while a plate on the other side will appear to be falling. The horizontal data constrain the geocenter velocity because the geocenter velocity changes the horizontal component of velocity of sites in different places by different amounts; if the geocenter velocity is wrong, the plates will appear to be deforming in the sphere the plates are rotating in. The procedure of first adjusting for a particular rebound model and next minimizing deviations from plate rigidity yields a measure of the goodness of fit of the rebound model.

Estimating vertical rates relative to the center of mass (model "C.M"). The laser ranging satellites Lageos-1 and -2 orbit about the center of mass of the earth (solid earth, oceans, and atmosphere). Therefore satellite laser ranging yields estimates of the radial component of velocity of the laser stations. Velocity solution CSR96L01 [Eanes and Watkins, electronic communication, 1997] specifies directly the radial (vertical)

component of site velocity relative to the mass center. Fixing the center of mass yields a hard constraint on the geodetic velocity solution in that the translational component of the reference frame is not estimated from the data themselves. The translational velocity for the laser solution is zero; the best estimate of the radial (vertical) component of velocity of a laser site is equal to that in CSR96L01.

Estimating vertical rates relative to the geocenter assuming that the uplift, subsidence, and deformation of the plate interiors is negligible after correcting for glacial isostatic adjustment and that the velocity of the geocenter equals that of the center of mass over decades (model "Geoc.= C.M."). The center of figure of the earth (the geocenter) lies 1.2 km nearer southeast Europe than the center of mass assuming the degree one spherical harmonic coefficients best fitting the topography of the solid earth (excluding oceans, ice sheets, and lakes) of Pavlis and Rapp [1990]. This offset is 0.02% of the radius of the earth. Averaged over 10 million years, the time period over which plate tectonic processes persist, the rate of offset is 0.1 mm/yr. Thus the small offset between geocenter and center of mass is one reason to believe that the velocity between the geocenter and the center of mass is insignificant.

In the appendix we present calculations indicating that the velocity between the geocenter and the center of mass generated by a number of phenomena (postglacial rebound, eustatic sea level rise, ice sheet unloading, continental drift) is less than a few tenths of mm/yr. Seasonal fluctuations of a couple centimeters exist between the geocenter and the center of mass, but these average to about zero over the twenty year period of observations [Watkins and Eanes 1993, 1997; Kar 1997]. Using observations of variations in the atmosphere, ocean, and surface ground water, Dong et al. [1997] calculate fluctuations between the geocenter and center of mass totaling several millimeters, and these fluctuations average to about zero over several years.

Estimating vertical rates relative to the geocenter assuming that the velocities of the plates equal those in NUVEL-1A and that the uplift, subsidence, and deformation of the plate interiors due to glacial isostatic adjustment is negligible (model "Geoc.-NUVEL1A"). The reference frame yielding no-net-rotation of the plates (NNR-NUVEL1A) [Argus and Gordon, 1991] is the frame in which site velocities are described in Goddard Space Flight Center's latest VLBI solutions (GLB1014j and GLB1083c [Ma and Ryan, electronic communication 1996, 1997]). The final step in the determination of

the International Terrestrial Reference Frame (ITRF) is transforming by the rotational and translational velocities that minimizes differences with NNR-NUVEL1A [Boucher et al. 1996, 1998; Z. Altamimi, oral communication 1996].) Therefore site velocities described in the ITRF are in the reference frame yielding no net rotation of the plates as given by NNR-NUVEL1A. The ITRF is the standard reference frame in which GPS velocities are described.

The ITRF [Boucher et al. 1996, 1998] is defined by minimizing differences between three-dimensional velocity observables and NNR-NUVEL1A. Minimizing differences between vertical rate observables and NNR-NUVEL1A assumes that vertical site rates are zero. The uplift and subsidence generated by glacial isostatic adjustment violates this assumption. Goddard Space Flight Center [Ma and Ryan, electronic communication 1996, 1997] defines the reference frame by minimizing differences between horizontal velocity observables and NNR-NUVEL1A assumes both that the velocities among the plates equal those in NUVEL-1A and that the velocities of the plates relative to the deep interior equal those in NNR-NUVEL1A. The latter assumption defines the rotational component of the reference frame. The former assumption defines the translational component of the reference frame and is probably wrong. Plate velocities averaged over several years estimated using geodesy are nearly equal to those averaged over a few millions of years estimated using spreading rates from magnetic anomalies, transform fault azimuths, and earthquake slip vectors [Robbins et al. 1993, Ma et al. 1994, Argus and Heflin 1995, Larson and Freymueller 1996]. But the plate velocities over the two time periods may differ by several percent for some plate pairs. In summary, Goddard's assumption that present-day plate velocities equal those predicted by NNR-NUVEL1A assumes too much, whereas the ITRF's assumption that the uplift and subsidence due to glacial isostatic adjustment is negligible is unrealistic.

Data Reduction

We estimate parameters treating the full data error matrix, that is we treat all correlations between the velocities of different sites.

We implement the various definitions of the reference frame by varying the data we invert and the parameters we estimate (Table 1). When we estimate velocities relative to the center of mass without assuming sites to be on plates, only data from sites with both SLR and VLBI constrain the reference frame. When we assume sites to be on plates,

only sites on plates constrain the reference frame. (In these models we do not impose velocity ties between sites not on plates with both VLBI and SLR.) When we assume sites to be on plates, we furthermore define the rotational velocity of the reference frame by fixing one of the plates. The models are invariant with regard to which plate is fixed.

In the descriptions in Table 1 we omit data from sites not on plates. When we add the velocity of a site not on a plate, we also introduce as a parameter the velocity of the site, so that the data is fit exactly by the parameter. Adding the velocities of sites not on plates simply allows us to estimate horizontal site velocities relative to plates and vertical rates of site motion relative to the geocenter.

We take the estimated uplift rate of a site on a plate to be the residual of the model after adding back the postglacial rebound prediction we subtracted.

Data. We combine VLBI solution GLB1083c [C. Ma and J. Ryan, Goddard Space Flight Center, electronic communication, 1997] and SLR solution CSR96L01 [R. Eanes and M. Watkins, Center for Space Research, University of Texas at Austin, electronic communication, 1997]. VLBI solution GLB1083c consists of the velocities of 82 sites and errors in and correlations between the site velocities, and is determined from interferometric data from November 1979 to July 1997. Nearly all the site velocities are of high quality. Seventy-five site velocities are determined from four or more data in three or more calendar years over a time period two years or longer. Seventy-three site velocities have one-dimensional standard errors in the horizontal components of site velocity less than 1.5 mm/yr. Thirty-nine vertical rates have standard errors less than 1 mm/yr. Twenty-five sites have data over eight years or longer.

The quality of the SLR velocity estimates vary widely. SLR solution CSR96L01 consists of the velocities of 72 laser ranging stations and errors in and correlations between the site velocities, and is determined from laser data from May 1976 to February 1996. Nine sites have vertical rates of motion with standard errors of 1 mm/yr or less, three sites have vertical standard errors between 1 and 2 mm/yr, and the remaining 60 sites have vertical errors greater than 2 mm/yr. Eleven site velocities have one-dimensional standard errors in horizontal components of site velocity less than 1 mm/yr, nine have horizontal errors between 1 and 2 mm/yr, and the remaining 52 sites have horizontal errors greater than 2 mm/yr. The sites with the smallest vertical errors are, from best to worst, Yaragadee (Australia), Monument Peak (California), Quincy (California), Maui

(Hawaii), Greenbelt (Maryland), Royal Greenwich Observatory (England), Grasse (France), Graz (Austria), and Macdonald Observatory (Texas). The median time span of data at these nine sites is 15 years. Three SLR vertical rates are anomalous, -5.2 ± 0.5 mm/yr (Maui), -9 ± 1.6 mm/yr (Orroral, Australia), and -28 ± 3.7 mm/yr (Haystack, Massachusetts). We treat these three vertical rates as outliers, that is, we omit them. Two of the three outliers are suspect for independent reasons. Haystack's huge subsidence is estimated from only two data points. Maui's velocity is estimated differently than all the others in that biases as a function of range were estimated.

Assignment of sites to plates. We assign 29 VLBI and 14 SLR sites to one of eight plates (Table 1). Sites are assigned to plate on the basis of major Holocene faulting, large historical earthquakes, seismicity, and topography following the criteria of Argus and Gordon [1996]. We take the western limit of the stable interior of the North American plate to be the boundary between the Great Plains and the Rocky Mountains. The VLBI data suggest that three sites on the Colorado Plateau are moving westward relative to the North American interior at 1 or 2 mm/yr [Argus and Gordon, 1996], with east-west extension across the Rio Grande Rift taking up the relative motion. We do not assign these three sites, Fairbanks (Alaska), or Penticton (British Columbia) to a plate. Nor do we assign the VLBI site at Fort Davis (Texas) or the SLR site at Macdonald Observatory (Texas) to the North American plate. The two sites lie in the Mexican Highland province of the southern Basin and Range, which may be taking up minor east-west extension associated with the Rio Grande Rift to the north.

Error budget. The formal errors in the VLBI solutions are undoubtedly overoptimistic [Ryan et al. 1993, Argus and Gordon 1996]. To achieve a realistic error budget we incorporate additional systematic error following the method of Argus and Gordon [1996]. For the VLBI data we take this systematic error to be inversely proportional to the time period of observations at a site (Table 3). For the SLR data we take the systematic error to be inversely proportional to the square root of the time period of observations at a site. The systematic error we incorporate is 2.5 to 3 times as large for the vertical as for the horizontal data. The error budget results in normalized misfits that are slightly less than 1 for each of four data subgroups (Table 3). Hence the errors are slightly conservative. The size of the horizontal error budget is determined roughly from the degree of consistency with plate rigidity after correcting for the horizontal

velocities predicted by Peltier [1994]. The size of the vertical error budget is determined roughly from the degree of consistency with the vertical rates predicted by Peltier [1994] or Peltier [1996].

Data importances. We compute the importance of each velocity input using equation 19 of Minster et al. [1974] (Table 1). The importance of a datum yields an estimate of what fraction of a parameter the datum is constraining. The data importances sum to the number of estimated parameters.

Results

Estimates of the Velocity of the Geocenter and That of The Center of Mass

We next examine estimates of the velocity of the geocenter and the center of mass relative to the VLBI and SLR tracking sites. These estimates are determined from inversions differing by constraints imposed on the translational velocities of the VLBI and SLR reference frames (Table 1).

We plot the three components of the velocity of the geocenter and center of mass in the special coordinate system defined by the principal axes of the error ellipsoid describing uncertainty in the velocity of the geocenter relative to the VLBI network. The minimum, intermediate, and maximum principal axes of the error ellipsoid are parallel to geocentric vectors to "A", "B", and "C", respectively (Figure 1). The great circle along the surface of the earth containing "A" and "B" passes through western Europe and eastern North America, which are the two places with the best constrained VLBI vertical rates. The components of the velocity of the geocenter and the center of mass in the plane containing "A", "B", and the geocenter has the biggest effect on the interpretation of vertical rates of site motion in western Europe and eastern North America, and it is for this reason that we plot the velocity estimates in the special coordinate system.

Estimates of the velocity of the geocenter and that of the center of mass as seen from the VLBI network. Imposing velocity ties at the 12 places with both techniques without assuming sites to be on plates (model "C.M.") yields a poorly constrained estimate of the velocity of the center of mass relative to the VLBI network (Figure 6, navy circle, navy 95% confidence ellipse). The 95% confidence limits in the estimate span a few mm/yr. Thus not assuming that sites are on plates results in estimates of vertical rates of motion that are too poorly constrained to be useful for comparison with

postglacial rebound models if the error budget we assume is correct. Watkins et al.'s [1994] determination of the velocity of the geocenter relative to the center of mass is equivalent in assumption to our determination, but the small confidence limits they estimate from the formal errors are overoptimistic.

Assuming that the uplift, subsidence, and deformation of the plate interiors is negligible after correcting for rebound (model "Geoc.") yields an estimate of the velocity of the geocenter relative to the VLBI network. The estimate depends on which postglacial rebound model is corrected for. The estimate determined correcting for Peltier [1994] (Figure 6, pink square and dashed pink error ellipse) differs from that determined correcting for Peltier [1996] (green square and green dashed error ellipse) by 0.95 mm/yr, but by only 0.39 mm/yr in the A-B plane. The estimates are constrained moderately well in the A-B plane but are uncertain along the C axis.

Assuming that the uplift, subsidence, and deformation of the plate interiors is negligible after correcting for rebound and that the velocity of the geocenter and that of center of mass are equal (model "Geoc.= C.M.") yields a well constrained estimate of the velocity of the geocenter (= center of mass). The estimate depends slightly on which postglacial rebound model is corrected for. The estimate determined correcting for Peltier [1994] (Figure 6, pink pentagon and solid pink error ellipse) differs from that determined correcting for Peltier [1996] (green pentagon and solid green error ellipse) by 0.32 mm/yr. Either estimate of the geocenter velocity differs by several tenths of mm/yr from that in GLB1083c (at the origin) [Ma and Ryan, electronic communication, 1997]. The three-dimensional 95% confidence ellipsoid describing uncertainty in the velocity has semi-principal axes of length 0.52, 0.61, and 0.73 mm/yr.

Assuming that the plates are rigid and that plate velocities equal those in NUVEL-1A while neglecting the uplift, subsidence, and deformation of the plate interiors (model "Geoc.-NUVEL1A") yields an estimate of the velocity of the geocenter that is very tightly constrained but probably wrong (Figure 6, gray square, gray error ellipse). The uplift and subsidence generated by postglacial rebound violates the assumption, as would any differences between plate velocities averaged over years and those averaged over millions of years. The geocenter velocity we estimate using these assumptions differs from that which Ma and Ryan [electronic communication, 1997] estimate using the same assumptions because we estimate the velocity of the geocenter using

horizontal and vertical data at 43 sites whereas they estimate it using horizontal data at nine sites.

Estimates of the velocity of the geocenter and center of mass as seen from the SLR network. Assuming that the uplift, subsidence, and deformation of the plate interiors is negligible after correcting for rebound (model "Geoc.") yields an estimate of the velocity between the geocenter and the center of mass. The estimate depends on which post-glacial rebound model is corrected for, that of Peltier [1994] (Figure 7, pink square, pink 95% confidence ellipse) or Peltier [1996] (green square, green error ellipse). Either estimate differs insignificantly from zero, that is the 95% confidence limits include the velocity of the center of mass (which is at the origin). Therefore the data are not inconsistent with the assumption that the velocity of the geocenter and that of the center of mass are equal. The estimate of the velocity of the geocenter relative to the center of mass correcting for Peltier [1994] is 1.5 ± 1.7 mm/yr toward the location along earth's surface at $70.4^\circ\text{S } 18.8^\circ\text{E}$, whereas the estimate correcting for Peltier [1996] is 2.0 ± 2.0 mm/yr toward the location along earth's surface at $58.1^\circ\text{S}, 40.6^\circ\text{W}$.

Caution is required when interpreting the estimates of speed because they are biased away from zero. For example, if the true velocity were zero, the estimated speed would always be greater than zero. We use Monte Carlo simulation to assess this tendency and to determine the best unbiased estimate of how fast the geocenter may be moving relative to the center of mass. We follow the method of Argus and Gordon [1996]. We later also use the Monte Carlo method to bound how fast a site may be moving relative to the plate on which it is assumed to lie. We explain how we determine unbiased best estimates and confidence limits in the caption of Figure 11. The unbiased estimate of the speed is -0.15 mm/yr correcting for Peltier [1994] and 0.5 mm/yr correcting for Peltier [1996]. The negative estimate indicates that assuming a true speed of zero and the estimated error ellipsoid yields an apparent speed greater than that estimated for over half of the Monte Carlo simulations. The unbiased one-sided upper 95% confidence limit is 3.0 mm/yr correcting for Peltier [1994] and 3.9 mm/yr correcting for Peltier [1996]. Insofar as the models of Peltier [1994] and Peltier [1996] represent the range of possible nonrigidity of the plates, the data limit the velocity of the geocenter relative to the mass center to be less than 4 mm/yr over the 20 year period of observations.

Assuming that the plates are rigid and that plate velocities equal those in NUVEL-1A while neglecting the uplift, subsidence, and deformation of the plate interiors (model "Geoc.-NUVEL1A") tightly constrains the velocity of the geocenter relative to the center of mass to a velocity differing significantly from zero (Figure 7, gray square, gray error ellipse). The estimate is determined using assumptions that are probably wrong.

Vertical Motions Due to Glacial Isostatic Adjustment

We next present estimate of vertical rates of site motion relative to the geocenter and center of mass from the inversion in which we assume the velocity of the two are equal (model "Geoc= C.M."). The VLBI vertical estimates depend slightly on which post-glacial rebound model is corrected for when defining the reference frame, whereas the SLR vertical estimates are invariant with respect to which model is corrected for. Uplift rates in eastern North American and western Europe are 0.2-0.3 mm/yr faster when defining the frame by correcting for Peltier [1996] as opposed to Peltier [1994] (compare left- and right-hand sides of Figure 8).

The observed and predicted vertical rates are highly correlated at the sixteen sites on plates with eight years or more of data (Figure 8). The weighted root means square of the misfit between observed and predicted rates is 1.1 mm/yr. Most data are misfit by more than the formal standard error and some data are misfit by a few times the formal standard error. The misfits are greater than those in a Gaussian distribution, indicating the formal standard errors are overoptimistic.

Rates of uplift are observed to decrease going away from the centers of the ancient Laurentide and Fennoscandian ice sheets (Figure 9). (We plot in Figure 9 the mean of the VLBI vertical rates determined correcting for Peltier [1994] and Peltier [1996].) The realistic 95% confidence limits in observed vertical rates computed from the modified error budget include the postglacial rebound model prediction at every site but Onsala (Figure 9). These 95% confidence limits are relative to the geocenter and are ± 2 to ± 3 mm/yr for most sites. The uncertainty in the relative rate of uplift between sites on the same continent may be smaller.

The model of Peltier [1994] predicts faster subsidence around the periphery of the ancient Laurentide ice sheet than does the model of Peltier [1996], and the axis of maximum subsidence lies nearer the ice sheet center in Peltier [1994] than in Peltier [1996] (Figures 1 and 9). The VLBI data tightly limit the gradient in uplift (to subsidence) rate

going away from the center of the Laurentide ice sheet in eastern North America. The site at Algonquin Park (Ontario) is rising at 2 mm/yr in response to unloading of the Laurentide ice sheet. (We quote uplift and subsidence rates to the nearest 0.5 mm/yr.) Haystack (Massachusetts) is neither rising nor falling, Westford (Massachusetts) is falling at 1 mm/yr, Greenbelt (Maryland) is falling at 1 mm/yr, and Green Bank (West Virginia) is falling at 1.5 mm/yr. Thus the northeast coast of the U.S. is observed to be subsiding slowly, suggesting that collapse of the ancient forebulge surrounding the Laurentide ice sheet is presently minor. The observed gradient in uplift (to subsidence) between Algonquin Park and Green Bank is nearer that predicted by Peltier [1996] than that predicted by Peltier [1994]. Richmond (Florida) is estimated to be falling at 0.5 mm/yr, agreeing with the minor subsidence rate predicted by either model.

The VLBI site at Fort Davis (Texas) is falling at 2 mm/yr and the SLR site at Macdonald Observatory (Texas) is falling at 3 mm/yr. These fast estimated subsidence rates disagree with the vanishing vertical rate expected far from the ice sheets. The observed subsidence may result from normal faulting associated with the Rio Grande Rift, but Fort Davis's slow horizontal velocity relative to the North American plate argues against significant rifting. The VLBI and SLR sites lie 8 km apart, making it unlikely for a local effect of spatial scale less than several km to be the cause of the fast subsidence rate.

Yellowknife's (Northwest Territories) estimated uplift rate, 8 ± 5.5 mm/yr (95% confidence limits), is highly uncertain because it is determined from data over only six years. The fast uplift rate suggests the ice sheet above Yellowknife was thicker during the last glacial maximum than in either Peltier [1994] or Peltier [1996]. Fairbanks (Alaska) is estimated to be rising at 1 mm/yr, whereas the two models predict minor subsidence. Fairbanks' estimated uplift rate may reflect Pacific-North America plate boundary effects, including a postseismic transient arising from the 1964 $M=8.5$ Prince William Sound earthquake.

The data also limit the gradient in uplift (to subsidence) rate going away from the center of the Fennoscandian ice sheet in Europe (Figure 9). Onsala (Sweden) is rising at 3.5 mm/yr in response to unloading of the Fennoscandian ice sheet. Effelsberg (Germany), Wettzell (Germany), Graz (Austria), and the Royal Greenwich Observatory (England) are falling at 0.5 to 1.5 mm/yr. This subsidence may reflect subsidence of the

ancient forebulge around the Fennoscandian ice sheet, but the estimated subsidence is faster than predicted by either Peltier [1994] or Peltier [1996]. The observation that Onsala is rising relative to these four sites at 4.5 mm/yr suggests that there is presently more "spring" left beneath the Fennoscandian ice sheet than predicted by either Peltier [1994] or Peltier [1996]. Additional "spring" would result from a thicker ice sheet above Fennoscandia during the last glacial maximum. More "spring" would also result from a higher upper mantle viscosity, which would produce a slower decrease in the rate of uplift over the time since the last glacial maximum. Madrid (Spain) is estimated to be rising at 2.5 mm/yr, whereas the two models predict minor subsidence.

In eastern North America and western Europe the uplift rates we estimate are 0.3 to 0.7 mm/yr faster than those in the original frame of Ma and Ryan [electronic communication 1997] (Figure 9, dashed squares). Insofar as our assumption that the velocity of the geocenter equals that of the center of mass over decades is true, and insofar as their assumption that current plate velocities equal those in NUVEL-1A is false, the vertical rates we estimate are more accurate than those estimated by Ma and Ryan [electronic communication 1997].

Horizontal Motions Due to Glacial Isostatic Adjustment

In North America the horizontal velocities predicted by the models of Peltier [1994] and Peltier [1996] differ significantly. The model of Peltier [1996] predicts moderately fast motion away from the center of the ancient Laurentide ice sheet, whereas the model of Peltier [1994] predicts very slow motion away from the center (Figure 2). Yellowknife and Algonquin Park, sites on opposite margins of the Laurentide ice sheet, are predicted to be moving apart at 3.6 mm/yr by Peltier [1996] and at 1.1 mm/yr by Peltier [1994]. The observed separation at 0.6 ± 2.0 mm/yr excludes Peltier [1996] but differs insignificantly from Peltier [1994]. (We consider differences significant if they differ at the 95% confidence level.)

Except for at Yellowknife, the horizontal velocities predicted by Peltier [1994] and Peltier [1996] are hard to distinguish. The horizontal velocities predicted in eastern North America differ considerably between the two models but the predicted intraplate deformation rates do not, and the data are sensitive to only the latter. For example, Algonquin Park and Richmond are predicted to be converging at 1.6 mm/yr by Peltier [1996] and at 0.9 mm/yr by Peltier [1994]. The observed convergence rate of 1.1 ± 1.4

mm/yr differs insignificantly from either model.

We distinguish between motions due to glacial isostatic adjustment and motions due to plate tectonics by interpreting only intraplate deformation as being generated by glacial isostatic adjustment. We present estimates of the velocity of each site on a plate relative to the plate on which it is assumed to lie (Figure 10, the black dashed ellipses are the 95% confidence limits in velocity estimated without correcting for postglacial rebound). We estimated these residuals by inverting the data many times, each time eliminating one site from its home plate [Argus and Gordon 1996]. The results show that the plates are to a high degree rigid. The velocities of all sites assumed to lie on the North American or Eurasian plates are less than 1.7 mm/yr except for the highly uncertain estimate at Potsdam. Furthermore all velocities differ insignificantly from zero except Onsala's. (That is the 95% confidence limits in velocity include the origin.)

We assume neither Fort Davis nor Fairbanks to be on the North American plate. Fairbanks' significant southward velocity relative to the interior of the North American plate is in the opposite direction of that of the Pacific plate and is probably due to a postseismic transient arising from the $M=8.5$ 1964 Prince William Sound earthquake [Pollitz and Argus, ms. in preparation].

The VLBI and SLR data provide tight upper bounds on how fast any site may be moving relative to the rest of its plate. Speeds faster than 2 mm/yr are excluded (with 95% confidence) for nine of the sites, consisting of five North American plate sites and four Eurasian plate sites (Figure 11). The upper bounds are smaller than those of Argus and Gordon [1996] and much smaller than those of Dixon et al. [1996].

Four of the sites assumed to be on plates have velocities near the limits of the 95% confidence ellipses, these being Onsala, Algonquin Park, Effelsberg, and Madrid. Algonquin Park is estimated to be moving southward at 1.0 mm/yr, away from the center of the Laurentide ice sheet, at about the velocity predicted by Peltier [1994] but slower than the velocity predicted by Peltier [1996]. Onsala is estimated to be moving southwestward at 1.3 mm/yr, roughly away from the center of the Fennoscandian ice sheet, at about the velocity predicted by either Peltier [1994] or Peltier [1996]. Madrid's northwest velocity relative to the interior of the Eurasian plate is parallel to the velocity of the African plate and may reflect Africa-Eurasian plate interaction, perhaps suggesting that Madrid is not strictly part of the Eurasian plate.

Comparing the estimated and predicted velocities is useful but may be misleading because glacial isostatic adjustment may bias the estimate of the angular velocity of a plate as well as the horizontal velocity of a site. Correcting for the glacial isostatic adjustment predicted by a particular model and then computing the velocity of each site relative to the plate on which it is assumed to lie provides a better assessment of the models.

Correcting for the predictions of Peltier [1994] reduces the residual velocities at Algonquin Park and Onsala but hardly changes the residual velocities at the other sites on plates (Figure 10, green 95% confidence ellipses). Thus correcting for the model of Peltier [1994] reduces horizontal misfits.

Correcting for the predictions of Peltier [1996] increases the residual velocity at Yellowknife a lot (Figure 10, purple 95% confidence ellipses). Yellowknife's residual velocity is a statistically significant 3.8 ± 1.9 mm/yr. This large residual reflects the observation that Yellowknife is not moving away from eastern North America as predicted by Peltier [1996]. Eliminating Yellowknife from the North American plate after correcting for Peltier [1996] allows the part of the plate in eastern North America to move more northwesterly to compensate for the southeastward motion produced by glacial isostatic adjustment there, resulting in the large residual at Yellowknife.

Goodness of Fit of the Models of Peltier [1994] and Peltier [1996]

Data misfits of the plate model with no correction for postglacial rebound are largest at the sites with the fastest predicted rebound rates. Onsala ($\chi^2 = 26.7$), Algonquin Park ($\chi^2 = 9.4$), and Yellowknife ($\chi^2 = 9.4$) are the poorest fit data. Correcting for the model of Peltier [1994] reduces these three misfits by a factor of three, whereas correcting for the model of Peltier [1996] reduces the misfits of Onsala and Algonquin Park by a factor of three but increases the misfit of Yellowknife by a factor of two. That the data poorest fit by the plate model are precisely those with the fastest predicted glacial isostatic adjustment, and that these misfits are significantly reduced by correcting for the predictions strongly indicates that glacial isostatic adjustment is being observed. Correcting for the postglacial rebound model of Peltier [1994] reduces the misfits of the plate model by 34%, whereas correcting for the model of Peltier [1996] reduces misfits by 16% (Figure 12). The Peltier [1994] misfit reduction ($F = 1.523$) is significant at the 95% confidence level, whereas the Peltier [1996] misfit reduction ($F = 1.197$) is not.

The vertical data are fit about equally well by the models of Peltier [1994] and Peltier [1996]. Correcting for Peltier [1994] reduces vertical misfits by 44%, whereas correcting for Peltier [1996] reduces vertical misfits by 39%. Peltier [1996] better fits the slow estimated subsidence of Haystack, whereas Peltier [1994] better fits the fast estimated uplift of Onsala and Madrid. (Although the vertical predictions at Onsala and Madrid differ negligibly between Peltier [1994] and Peltier [1996], defining the reference frame by correcting for the former yields slightly faster uplift rates in western Europe (Figure 8), reducing misfits at Onsala and Madrid.)

The horizontal data are fit significantly better by Peltier [1994] than by Peltier [1996]. Correcting for Peltier [1994] reduces horizontal misfits by 18%, whereas correcting for Peltier [1996] increases horizontal misfits by 16%. The increase in misfit of Peltier [1996] is due entirely to the horizontal velocity at Yellowknife, which is not moving away from eastern North America as predicted. The reduction in misfit of Peltier [1994] is due to the horizontal velocities at Onsala and Algonquin Park, which are moving away from the centers of the ice sheets as predicted.

Conclusions

(1) The velocity of the geocenter defines the translational velocity of the reference frame in which site velocities are described in global space geodetic solutions. The velocity of the geocenter is usually estimated (and the translational velocity of the reference frame defined) assuming the velocities of the plates equal those in NUVEL-1A [DeMets et al. 1990, 1994] while neglecting the uplift, subsidence, and intraplate deformation produced by glacial isostatic adjustment. The uplift and subsidence generated by postglacial rebound violates this assumption, as would any differences between plate velocities averaged over years and those averaged over millions of years.

(2) We estimate the velocity of the geocenter assuming that the uplift, subsidence, and intraplate deformation of the interiors of the plates is negligible after correcting for glacial isostatic adjustment.

(3) We maintain it reasonable to assume that the velocity of the geocenter relative to the center of mass is negligible when averaged over decades because we can identify no phenomenon violating the assumption.

(4) The VLBI and SLR data provide a check of the assumption that velocity of the geocenter relative to the center of mass is negligible when averaged over decades. We estimate the velocity of the geocenter relative to the center of mass to differ insignificantly from zero. The data limit the velocity of the geocenter relative to the center of mass to less than 4 mm/yr. Therefore the data are not inconsistent with the assumption that the velocity of the geocenter equals that of the center of mass over decades.

(5) The VLBI and SLR data observe the isostatic adjustment of the solid earth in response to unloading of the late Pleistocene ice sheets. Correcting for the predictions of Peltier [1994] reduces the total misfit relative to the rigid plate model by a third. Correcting for the predictions of either Peltier [1994] or Peltier [1996] reduces vertical misfits by about 40%.

(6) Onsala (Sweden) is estimated to be rising at 3 mm/yr in response to unloading of the Fennoscandian ice sheet. Algonquin Park (Ontario) is estimated to be rising at 2 mm/yr in response to unloading of the Laurentide ice sheet.

(7) The data tightly limit the gradient in uplift (to subsidence) rate going away from the center of the Laurentide ice sheet in eastern North America. Four sites along the northeast coast of the U.S. are falling at less than 1.5 mm/yr, suggesting that collapse of the ancient forebulge around the Laurentide ice sheet is presently very slow.

(8) Onsala is rising at 4.5 mm/yr relative to four sites in western Europe, suggesting that there may be more "spring" left in the Fennoscandian ice sheet than predicted by Peltier [1994, 1996].

(9) Sites on the margins of the ancient ice sheets are observed to be moving away from their centers very slowly or not at all. This observation is consistent with the model of Peltier [1994] but inconsistent with the model of Peltier [1996], which predicts the ice sheet margins to be moving away from the centers at a few mm/yr.

(10) On the other hand the model of Peltier [1996] well fits the slow collapse of the Laurentide forebulge along the east coast of the U.S. evident in relative sea level histories (from radiocarbon dates), whereas the model of Peltier [1994] overpredicts the collapse rate. How might the model of Peltier [1996] be modified to fit both the slow forebulge collapse and the very slow outward motion of the ice sheet margins? The thickness of the elastic lithosphere might be increased, the radial profile of mantle viscosity might be altered, or lateral variations in mantle viscosity might be introduced. In any

case the observation that the margins of the ancient ice sheets are moving away from their centers very slowly or if at all places an important constraint on the deglaciation history and mantle viscosity assumed in postglacial rebound models.

(11) The long history of VLBI and SLR data from sites around the periphery of the ancient ice sheets provide a tightly constrained reference frame in which to describe vertical rates estimated from Global Positioning System (GPS). Comparing vertical rates of site motion determined using VLBI and SLR with those estimated using the growing network of GPS sites (i.e, the BIFROST [1996] sites across Fennoscandia) is important for assessing geodetic estimates of the vertical and horizontal motion presently produced by glacial isostatic adjustment.

Appendix

Calculations of Movement of the Mass Center Relative to the Geocenter Generated by Various Phenomena

Laurentide ice sheet rebound. The present-day uplift of Canada in response to unloading of the ancient Laurentide ice sheet produces a change in mass at the surface, where rock is replacing air. The uplift also produces changes in mass in the upper mantle, where more dense rock is replacing less dense rock.

Approximating the present-day uplift of Canada as a disk of radius 1112 km (10° along the earth's surface) and assuming rock of density 2700 kg/m^3 is replacing air of negligible density yields an increase in mass equal to $1.0 \times 10^{14} \text{ kg}$ per year, resulting in the center of mass moving toward Canada at 0.11 mm/yr .

Assuming that more dense rock is replacing less dense rock in the crust and upper mantle to the depth of the transition zone, and assuming a density gradient as a function of depth of 2.1 kg/m^3 per km [Turcotte and Shubert, 1982, appendix 1, table F] yields an increase in mass equal to $5.2 \times 10^{13} \text{ kg}$ per year, resulting in the center of mass moving toward Canada at 0.055 mm/yr . It is doubtful that the entire upper mantle is rising; lateral movement of rock also fills the void left by the rock rising at the surface; therefore our calculation is an upper bound.

Thus solid earth replacing air at the surface has a bigger effect than more dense rock replacing less dense rock in the crust and upper mantle.

1° along the earth's surface=

$$(\pi \text{ rad}/180^\circ) \times (6371 \times 10^3 \text{ m}) = 111.2 \times 10^3 \text{ m}$$

Rock replacing air at surface

mass increase=

$$(\text{Laurentide ice sheet area}) \times (\text{uplift rate}) \times (\text{rock density}) =$$

$$\pi [(10^\circ \times (111.2 \times 10^3 \text{ m})/1^\circ)^2 \times (.01 \text{ m/yr}) \times (2700 \text{ kg/m}^3)] = 1.0 \times 10^{14} \text{ kg}$$

$$(\text{movement of center of mass})/(\text{earth radius}) = (\text{mass increase})/(\text{earth mass})$$

movement of center of mass=

$$(6371 \times 10^3 \text{ m}) \times (1.0 \times 10^{14} \text{ kg}) / (6 \times 10^{24} \text{ kg}) = .00011 \text{ m}$$

Less dense replacing more dense rock in crust and upper mantle

density at surface is 2700 kg/m^3

density 650 km deep is 4050 kg/m^3

$$\text{mantle density gradient} = (1350 \text{ kg/m}^3) / (650 \times 10^3 \text{ m}) = 2.1 \times 10^{-3} \text{ kg/m}^3 / \text{m}$$

mass increase=

$$(\text{Laurentide ice sheet area}) \times (\text{uplift rate}) \times (\text{density gradient}) \times \text{depth} =$$

$$\pi [10^\circ \times (111.2 \times 10^3 \text{ m}/1^\circ)]^2 \times .01 \text{ m/yr} \times (2.1 \times 10^{-3} \text{ kg/m}^3/\text{m}) \times (650 \times 10^3 \text{ m}) = 5.2 \times 10^{13} \text{ kg}$$

movement of center of mass=

$$(6371 \times 10^3 \text{ m}) \times (5.2 \times 10^{13} \text{ kg}) / (6 \times 10^{24} \text{ kg}) = .000055 \text{ m}$$

movement of center of figure=

$$(\text{uplift rate}) \times (\text{Laurentide ice sheet area}) / (\text{earth surface area}) =$$

$$(.01 \text{ m/yr}) \times \pi [(10^\circ/90^\circ \text{a})^2 / (4/3\pi \text{a}^2)] = .01 \text{ m/yr} \times .0093 = .000093 \text{ m/yr}$$

Sea level rise caused by Antarctic ice sheet thinning. If the present-day rise of sea level at 2 mm/yr were due entirely to transforming ice at the bottom of the Antarctic ice sheet to ocean water, then the Antarctic ice sheet would be thinning at 44 mm/yr , a mass of $2.3 \times 10^{14} \text{ kg/m}^3$ per year would be transferred from the Antarctic ice sheet to the oceans, and the center of mass would be moving at 0.24 mm/yr toward the North pole. It is doubtful that thinning of the Antarctic ice sheet creates all of the present-day eustatic sea level rise; the thermal expansion of water in response to warming may cause a significant fraction of the sea level rise; therefore our calculation is an upper bound.

ice sheet thinning rate=

$$(\text{ocean surface area}/\text{ice sheet surface area}) \times \text{ocean rise rate} =$$

$$(2/3) \times [4/3 \times \pi \text{a}^2] / [\pi (18^\circ/90^\circ \text{a})^2] \times (.002 \text{ m/yr}) = .044 \text{ m/yr}$$

assuming that 2/3 of the surface of the earth is water,
approximating the Antarctic ice sheet as a circle with an 18° radius,
a is the radius of the earth.

Antarctic ice sheet mass decrease=

ocean mass increase= (ocean area) X (ocean rise rate) X (ocean density)=

$$(2/3 \pi a^2) \times (.002 \text{ m/yr}) \times (1000 \text{ kg/m}^3) = 2.3 \times 10^{14} \text{ kg/yr}$$

movement of center of mass=

$$(6371 \times 10^3 \text{ m}) \times (2.3 \times 10^{14} \text{ kg}) / (6 \times 10^{24} \text{ kg}) = .00024 \text{ m}$$

Continental drift. If a mountain of height 5 km, width 5000 km (45° along the earth's surface), and length 560 km (5° along the earth's surface) were moving at 100 mm/yr, then the center of mass would be changing at a minuscule 2.3×10^{-7} mm/yr.

Horizontal movements of mass at the surface of the earth produce much less movement of the center of mass than does vertical movements of mass. This is because the moment (distance from the center of mass in the direction of movement) is so much less for horizontal movements (it being nearly zero) than for vertical movements (when it is 1 earth radius).

mountain area= height X width X length=

$$(5 \times 10^3 \text{ m}) \times [5^\circ \times (111.2 \times 10^3 \text{ m/}^\circ)] \times [45^\circ \times (111.2 \times 10^3 \text{ m/}^\circ)] = 1.4 \times 10^{16} \text{ m}^3$$

plate rate= .1 m/yr

(movement of center of mass)/(continent movement)= (mass increase)/(earth mass)

$$\text{movement of center of mass} = (.1 \text{ m}) \times (1.4 \times 10^{16} \text{ kg}) / (6 \times 10^{24} \text{ kg}) = 2.3 \times 10^{-10} \text{ m}$$

Mantle convection beneath East Pacific Rise. The movement of the center of mass generated by the rise of the upper mantle beneath the East Pacific Rise is not easy to assess. At the spreading center lithosphere is being accreted to the plates on either side. The upper mantle may be rising at 100 mm/yr (roughly half the rate of separation between the two plates on either side of the spreading center), and the dimensions of the rock rising may be approximated by a rectangular parallelepiped 6670 km (60° along the earth's surface) long, 200 km wide (twice the thickness of the lithosphere), and 650 km deep (to the transition zone.)

Calculating the mass change for mantle convection differs from calculating it for postglacial rebound in that there is no place where rock is replacing air. In fact the

density of the mantle as a function of depth may not be changing at all in time, which would result in no mass change and no movement of the center of mass. The calculation we give next is unrealistic in that it neglects the fact that the density of rock decreases as it rises, but it nevertheless gives an idea of the variables involved.

Less dense replacing more dense rock in crust and upper mantle

mass increase= East Pacific Rise area X uplift rate X density gradient X depth=

$$(60^\circ \times (111.2 \times 10^3 \text{ m}/1^\circ) \times (200 \times 10^3 \text{ m}) \times (.1 \text{ m/yr}) \times (2.1 \times 10^{-3} \text{ kg/m}^3 / \text{m}) \times (650 \times 10^3) = 1.8 \times 10^{14} \text{ kg}$$

movement of center of mass=

$$(6371 \times 10^3 \text{ m}) \times (1.8 \times 10^{14} \text{ kg}) / (6 \times 10^{24} \text{ kg}) = .00019 \text{ m}$$

World's population to Antarctica. If all the people on earth were to move to Antarctica next year, then the mass in Antarctica would increase by $3.8 \times 10^{11} \text{ kg}$, moving the center of mass toward Antarctica at $4.0 \times 10^{-4} \text{ mm/yr}$.

Mass movements by man tend to be short relative to the radius of the earth, so it is doubtful that these would move the center of mass significantly either.

$$\text{mass increase in Antarctica} = (5 \times 10^9 \text{ people}) \times (75 \text{ kg/person}) = 3.8 \times 10^{11} \text{ kg}$$

$$\text{movement of center of mass} = (6371 \times 10^3 \text{ m}) \times (3.8 \times 10^{11} \text{ kg}) / (6 \times 10^{24} \text{ kg}) = 4.0 \times 10^{-7} \text{ m}$$

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Figure captions

Figure 1. Rates of uplift and subsidence predicted by the postglacial rebound models of Peltier [1994] and Peltier [1996] are compared. The locations of VLBI (squares) and SLR (circles) sites are shown. (white-filled symbols are sites assumed to be on plates, black-filled symbols are sites not assumed to be on plates.) Uplift rates reach a local maximum in Hudson Bay, at the center of the Laurentide ice sheet, and in the Gulf of Bothnia, at the center of the Fennoscandian ice sheets. Peltier [1994] predicts faster rates of subsidence than Peltier [1996] around the periphery of the Laurentide ice sheet (the red shades are deeper and more widespread in the top diagram).

Vertical rates along the line segments labeled "L-L" and "F-F" are plotted in Figure 8. Geocentric vectors to "A", "B", and "C" are parallel to the minimum, intermediate, and maximum principal axes of the error ellipsoid describing uncertainty in the velocity of the geocenter relative to the VLBI network. These directions define the coordinate system in which we plot estimates of the velocity of the geocenter in Figures 6 and 7.

Figure 2. Horizontal velocities in North America predicted by the postglacial rebound models of Peltier [1994] and Peltier [1996] are compared. Peltier [1996] predicts the margins of the Fennoscandian ice sheet to be moving away from the center moderately fast, whereas Peltier [1994] predicts the margins to be moving away from the center very slowly. See Peltier [1997] for the horizontal velocities in Europe predicted by the two models.

Figure 3. The viscosity of the mantle as a function of depth is compared between the models of Peltier [1994] and Peltier [1996]. Peltier [1994] assumes a simple two-layer model, whereas various geophysical data allow Peltier [1996] to estimate detailed variations in mantle viscosity.

Figure 4. Shown is a translational velocity along the geocentric vector to the North pole or, equivalently, a change in the velocity of the geocenter along the geocentric vector to the South pole. The change in the horizontal and vertical velocity components of a site depends on site location. At the North pole the translational velocity shown increases uplift, whereas at the equator it increases the north component of velocity.

Figure 5. Schematic diagram showing the effect of the postglacial rebound in response to unloading of the Laurentide ice sheet on estimates of the velocity of the geocenter. Many sites (crosses) are on the North American plate, but only one site is on the

Antarctic plate. The short light blue line segments show the limits of the plates.

We define the velocity of the geocenter (black circle) to be that which yields no uplift or subsidence of the plate interiors after removing the effect of glacial isostatic adjustment. If postglacial rebound were the only process by which the earth deforms, the velocity of the geocenter we define would equal the velocity of the deep interior of the earth because the uplift associated with postglacial rebound does not extend into the lower mantle.

In the diagram the velocity of the geocenter differs slightly from that of the center of figure. Approximating the present-day uplift of Canada as a disk with a 1112 km radius (10° along the surface of the earth) rising at 10 mm/yr, we estimate the velocity of the geocenter relative to the center of figure to be 0.1 mm/yr (see calculation in appendix).

If we were to estimate the velocity of the point yielding no uplift or subsidence of the plate interiors without correcting for glacial isostatic adjustment, we would determine a biased estimate of the velocity of the deep interior of the earth. If one-quarter of the sites on the North American and Antarctic plates used to estimate this biased reference point were rising at 10 mm/yr due to Laurentide postglacial rebound, the estimate of the velocity of the center point would be biased by roughly 2.5 mm/yr ($=1/4 \times 10$ mm/yr). Sites not on the North American or Antarctic plates are poor indicators of the component of the velocity of the geocenter toward Laurentia because it is difficult to determine the plate motion contribution to this component of a site's velocity.

Figure 6. Estimates of the velocity of the geocenter and that of the center of mass as seen from the VLBI tracking network are compared. Error ellipses are 95% confidence limits. At the origin is the velocity of the geocenter in GLB1083c [Ma and Ryan, electronic communication 1997]. The coordinate system is defined by the minimum ("A"), intermediate ("B"), and maximum ("C") principal axes of the error ellipsoid describing uncertainty in the velocity of the geocenter relative to the VLBI network (see Figure 1).

The estimates of the velocity of the geocenter and the center of mass are determined using different assumptions. Not assuming sites to be on plates poorly constrains the velocity of the center of mass ("C.M.", navy square, navy 95% confidence ellipse.) Assuming plate velocities to be equal to those in NUVEL-1A while neglecting the uplift, subsidence, and deformation of the plates is unrealistic but would tightly constrain the velocity of the geocenter ("Geoc.-NUVEL1A", gray square, gray 95% confidence

ellipse). Assuming that the uplift, subsidence, and deformation of the plates equal those predicted by a model of glacial isostatic adjustment constrains the velocity of the geocenter moderately well ("Geoc.", the pink square and dashed pink 95% confidence ellipse assumes the model of Peltier [1994], whereas the green square and dashed green 95% confidence ellipse assumes the model of Peltier [1996]). Assuming that the velocity of the geocenter relative to the center of mass is negligible when averaged over decades and that the uplift, subsidence, and deformation of the plates equal those predicted by a model of glacial isostatic adjustment well constrains the velocity of the geocenter ("Geoc.=C.M.", the purple hexagon and solid purple error ellipse assumes the model of Peltier [1994], whereas the green hexagon and solid green error ellipse assumes the model of Peltier [1996]).

Figure 7. Estimates of the velocity of the geocenter and that of the center of mass as seen from the SLR tracking network are compared. Error ellipses are 95% confidence limits. At the origin is the velocity of the center of mass [Eanes and Watkins, electronic communication 1997]. The coordinate system is defined by the minimum ("A"), intermediate ("B"), and maximum ("C") principal axes of the error ellipsoid describing uncertainty in the velocity of the geocenter relative to the VLBI network (see Figure 1).

The estimates of the velocity of the geocenter relative to the center of mass are determined using different assumptions. Assuming plate velocities to be equal to those in NUVEL-1A while neglecting the uplift, subsidence, and deformation of the plates is unrealistic but would tightly constrain the velocity of the geocenter ("Geoc.-NUVEL1A", gray square, gray 95% confidence ellipse). Assuming that the uplift, subsidence, and deformation of the plates equal those predicted by a model of glacial isostatic adjustment constrains the velocity of the geocenter moderately well ("Geoc.", the pink square and dashed pink 95% confidence ellipse assumes the model of Peltier [1994], whereas the green square and dashed green 95% confidence ellipse assumes the model of Peltier [1996]).

Figure 8. Predicted rates of vertical site motion are compared with those estimated assuming that the uplift, subsidence, and deformation of the plate interiors is negligible after correcting for rebound and that the velocity of the geocenter equals that of the center mass over decades. VLBI vertical estimates determined defining reference frame correcting for Peltier [1994] (four diagrams on the left) differ slightly from VLBI vertical

estimates determined defining reference frame correcting for Peltier [1996] (four diagrams on the right). Uplift rates in eastern North American and western Europe are 0.2-0.3 mm/yr faster when defining frame correcting for Peltier [1996] as opposed to Peltier [1994]. SLR vertical estimates are invariant with respect to which model is corrected for. All estimates of vertical rates of sites on plates with eight years or more of data are shown on the top pair of diagrams. The bottom three pairs of diagrams show these same vertical rate estimates segregated by plate.

If the data were exact and the models were perfect all the data would lie along the 45 degree lines. The correlation coefficient ("r") computed from the data from sites on plates show that the observed and predicted vertical rates are highly correlated at the sixteen sites on plates with eight years or more of data. The weighted root means square ("wrms") of the misfit between observed and predicted rates is 1.1 mm/yr.

Figure 9. Observed and predicted vertical rates of site motion are compared along line segments across the ancient Laurentide (top) and Fennoscandia (bottom) ice sheets. All vertical rates shown are determined from data over eight years or longer except for the vertical rate at Yellowknife, which is determined from data over six years. The angular distance from the ice sheet center is plotted along the horizontal axis. The 95% confidence limits computed using the error budget described in the text are realistic. The VLBI vertical rates plotted are the mean of estimates computed correcting for Peltier [1994] and Peltier [1996], that is, the mean of the VLBI vertical rates on the left- and right-hand sides of Figure 8. Vertical rates of site motion in the original GLB1083c frame of Ma and Ryan [electronic communication, 1997] are also shown (faint gold squares) at selected sites.

Four sites along the northeast coast of the U.S. are falling at less than 1.5 mm/yr, suggesting that collapse of the ancient forebulge around the Laurentide ice sheet is presently very slow. Onsala is rising at 4.5 relative to four sites in western Europe, suggesting that there may be more "spring" left in the Fennoscandian ice sheet than predicted by Peltier [1994, 1996].

Figure 10. The 95% confidence limits in the horizontal velocity of each site on the North American and Eurasian plate relative to the remaining sites on the plate estimated without correcting for postglacial rebound (black ellipses) are compared with the velocity predicted by Peltier [1994] (maroon "X"s) and by Peltier [1996] (green "Y"s). If

the predicted velocity is less than a half mm/yr it is omitted for clarity. The 95% confidence limits in residual velocity include the origin at every site except Onsala, indicating the horizontal velocity presently generated by glacial isostatic adjustment is small. The largest residual velocities are at Onsala, Algonquin Park, Effelsberg, and Madrid. Onsala's estimated velocity agrees with the prediction from either Peltier [1994] or Peltier [1996], whereas Algonquin Park's velocity agrees with the prediction of Peltier [1994] but not Peltier [1996]. Fairbanks and Fort Davis are not assumed to be on the North American plate.

Correcting for the predictions of Peltier [1994] before estimating the 95% confidence limits (pink ellipses) reduces the residual velocity at Onsala and Algonquin Park, suggesting that these sites, which are along the margins of the Fennoscandia and Laurentide ice sheets, are moving slowly away from the centers of the ice sheets. Correcting for the predictions of Peltier [1996] before estimating the 95% confidence limits (green ellipses) increases the residual velocity at Yellowknife, indicating that Peltier's [1996] model poorly fits Yellowknife's estimated velocity.

Figure 11. Speeds and confidence limits between sites and the plates on which they are assumed to lie. The apparent speed (open circle) estimated from the magnitude of the velocity in Figure 10 is biased upward, away from zero, as is the upper 95% confidence limit taken from the farthest point ("X") along the 95% confidence limits in this apparent velocity. The unbiased estimate of the true speed of a site (filled circle) is the hypothetical true speed that gives an expected speed equal to the observed apparent speed. In the cases in which no value of the hypothetical true speed is small enough to give an expected speed equal to that observed, the expected value for the case of zero true speed is subtracted from the observed speed to give an unbiased estimate of the true speed. The one-sided upper 95% confidence limit (short vertical line at the right-hand end of the error bar) is the hypothetical true speed for which only 5% of the realizations equal or are exceeded by the observed apparent speed. Two-sided (dashed) 95% confidence limits are also shown for Onsala, the sole site for which these limits exclude zero. The lower two-sided 95% confidence limit is the hypothetical true speed for which only 2.5% of the realizations equal or exceed the observed apparent speed.

Figure 12. The misfit (χ^2) of the plate model with no correction for glacial isostatic adjustment is compared with the misfits of the plate model correcting for Peltier [1994]

and Peltier [1996]. The misfits to the vertical ("V") and horizontal ("H") data are also shown. ("H" is actually the total misfit less the vertical misfit, which differs slightly from the horizontal misfit because of correlations between vertical and horizontal data.) Correcting for the postglacial rebound model of Peltier [1994] reduces the misfits of the plate model by 34%, whereas correcting for the model of Peltier [1996] reduces misfits by 16%. The Peltier [1994] misfit reduction is significant at the 95% confidence level, whereas the Peltier [1996] misfit reduction is not.

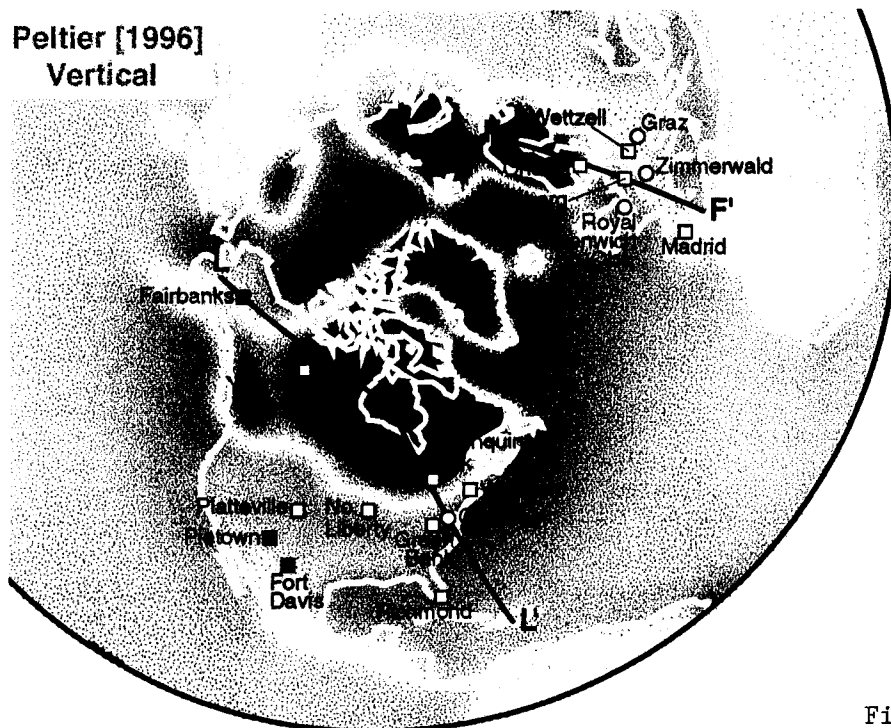
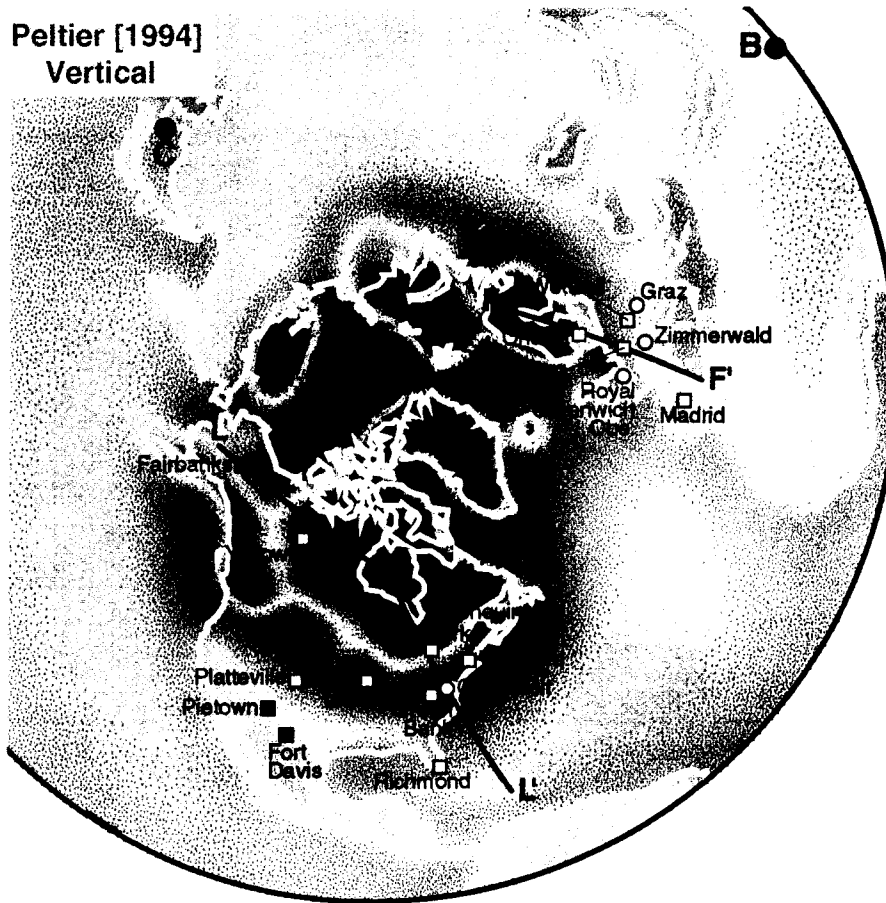
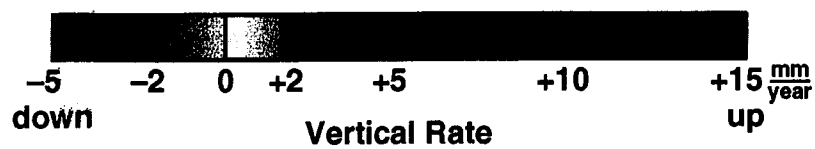
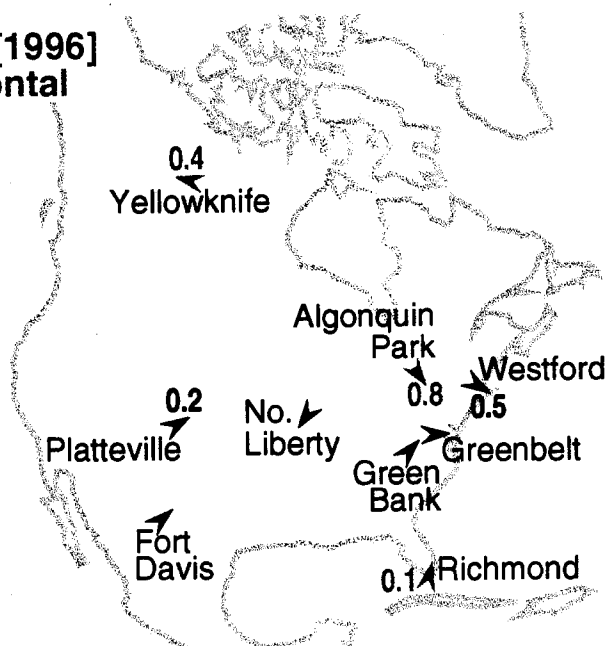


Figure 1



**Peltier [1996]
Horizontal**



**Peltier [1994]
Horizontal**

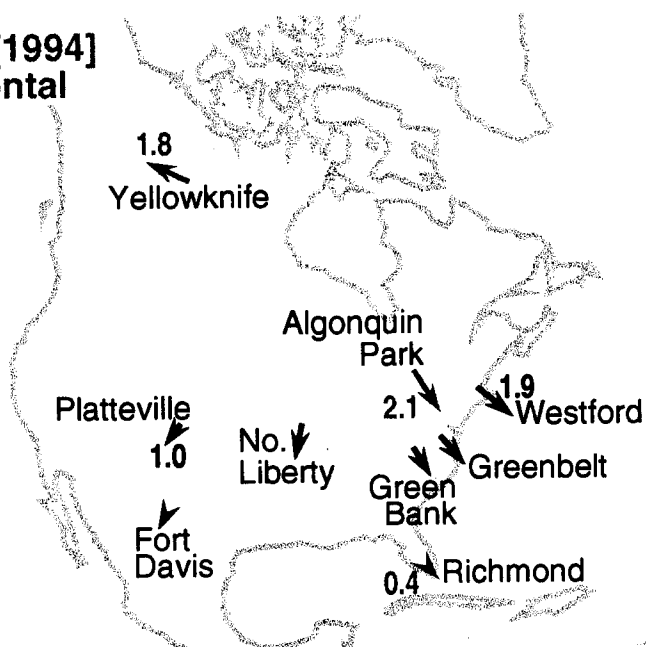


Figure 2

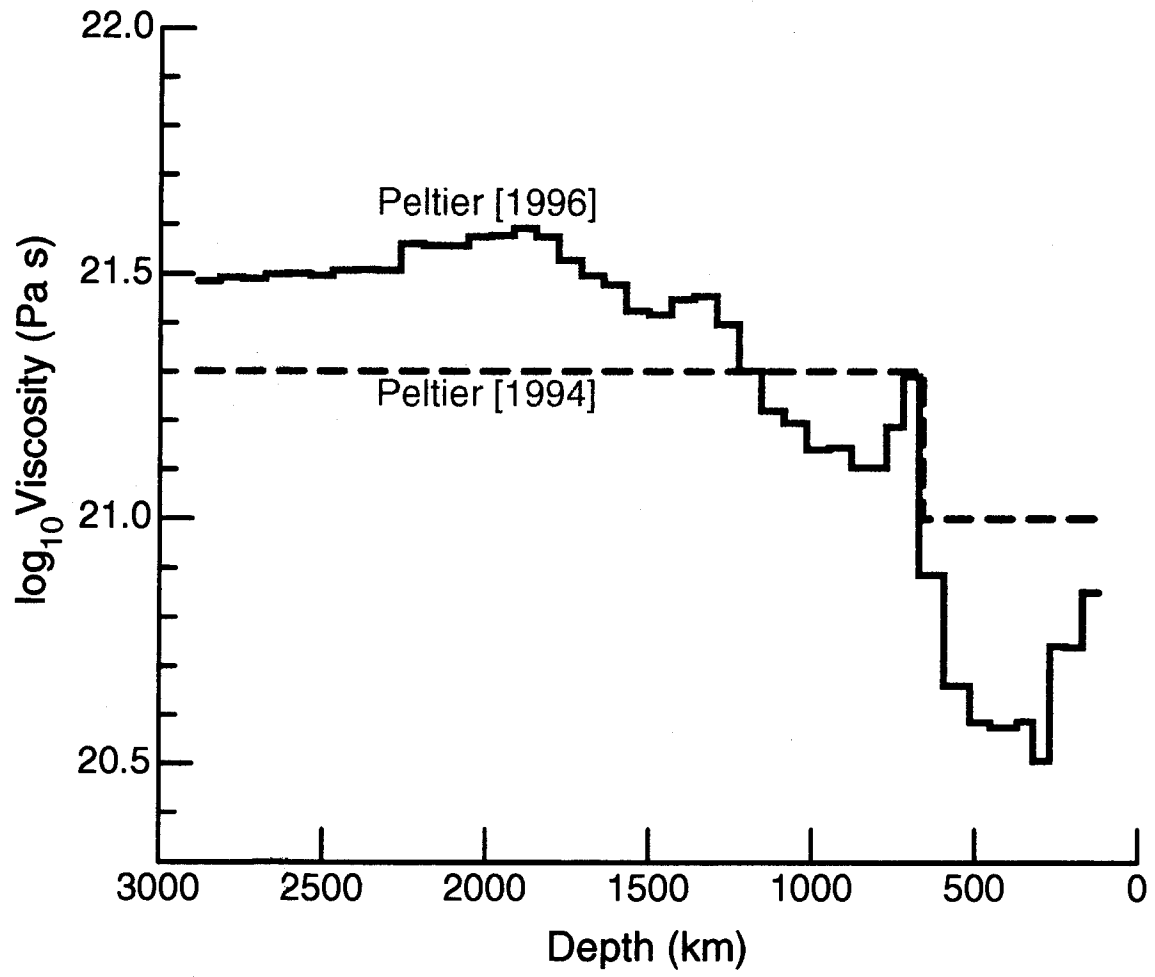


Figure 3

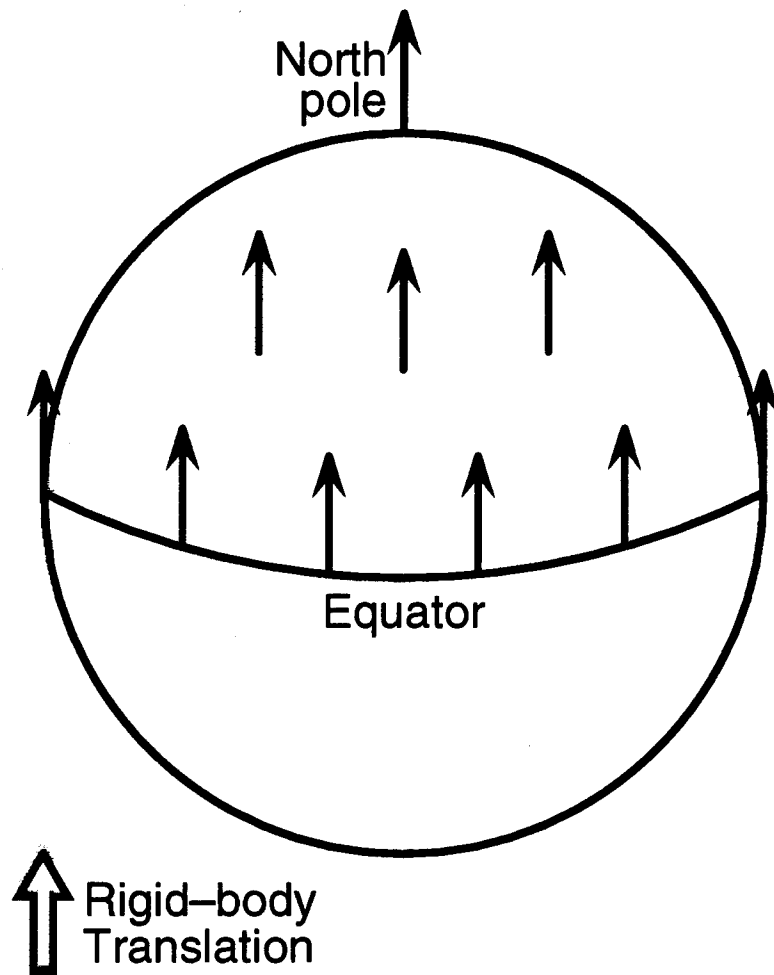


Figure 4

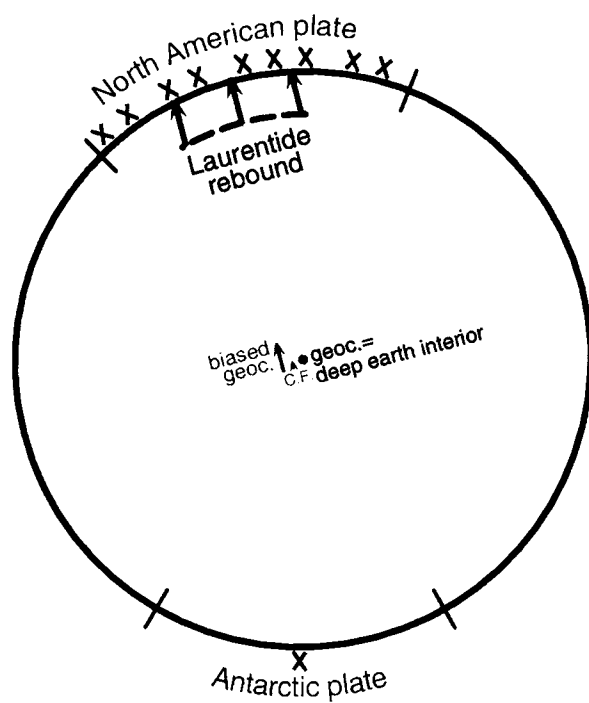
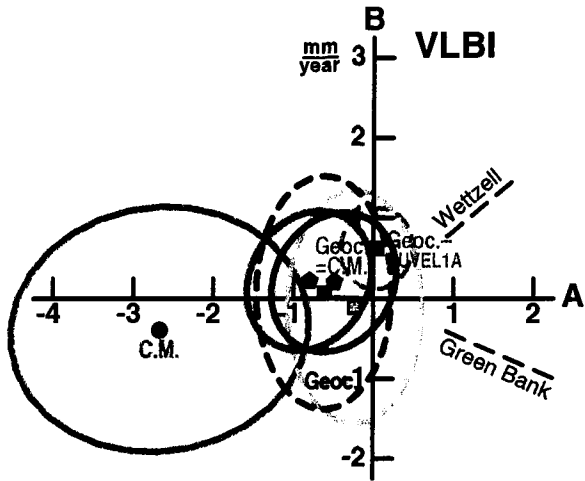


Figure 5



assuming
NUVEL-1A

Wetzell
Green Bank

correcting for
Peltier [1996]

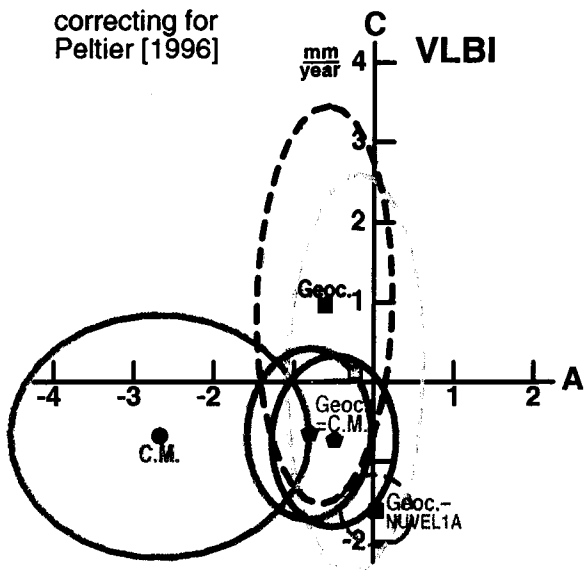
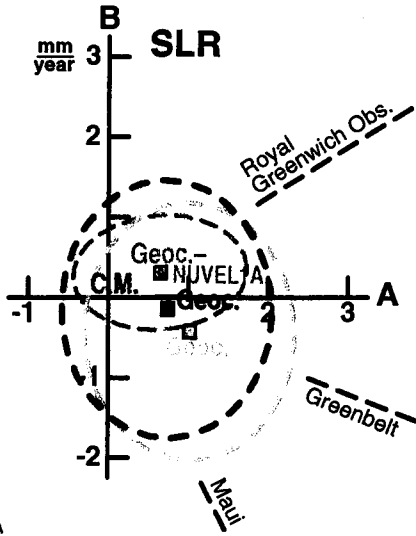


Figure 6



assuming
NUVEL-1A

correcting for
Peltier [1996]

correcting for
Peltier [1996]

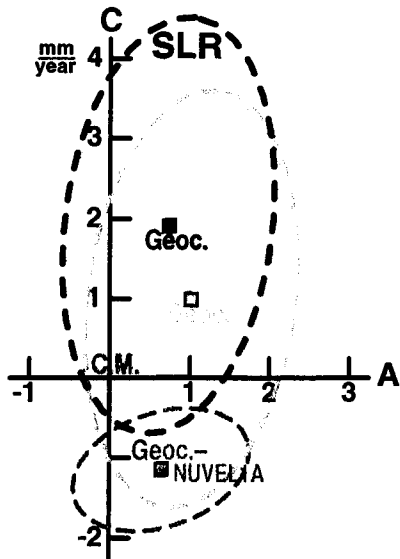


Figure 7

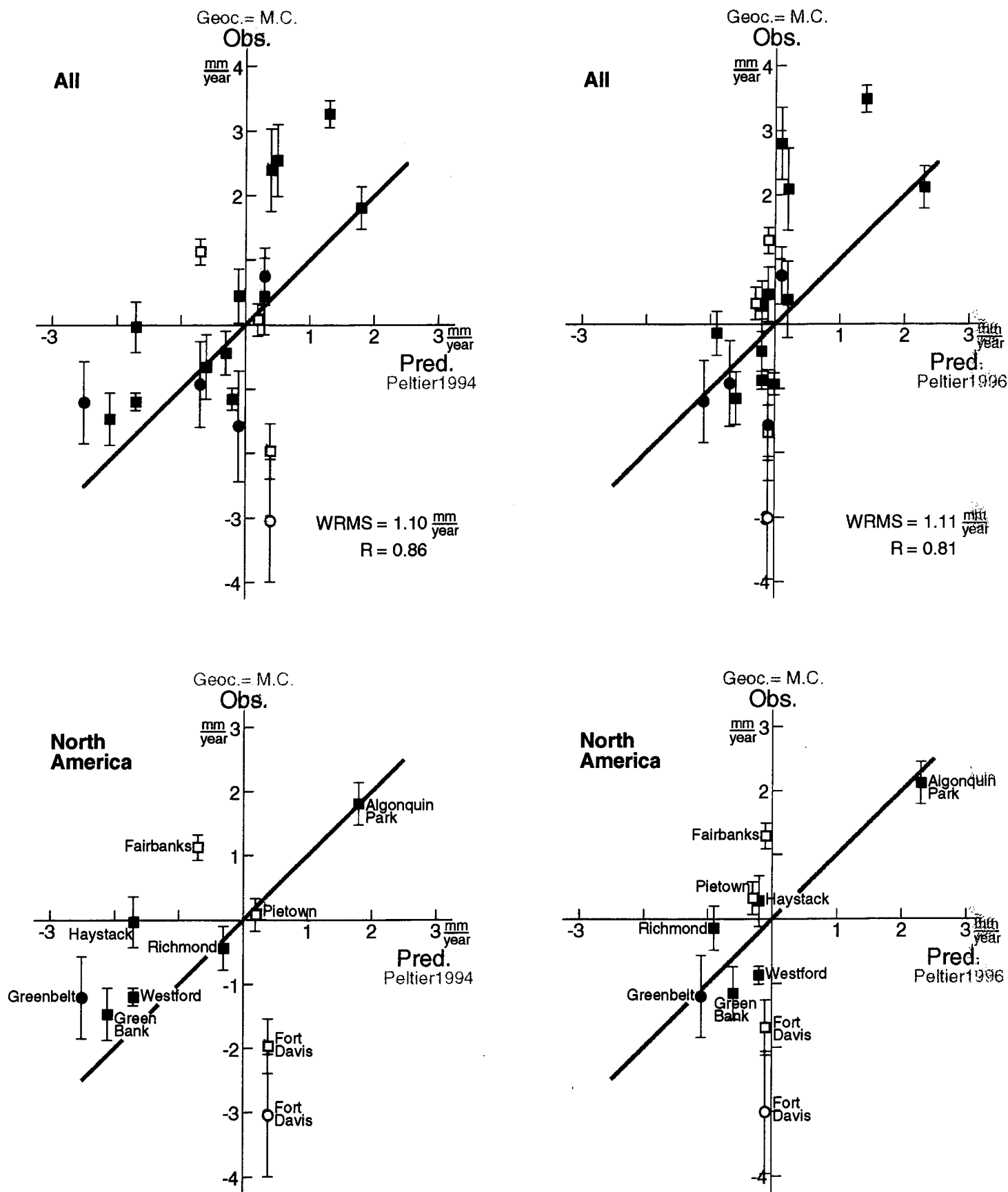


Figure 8

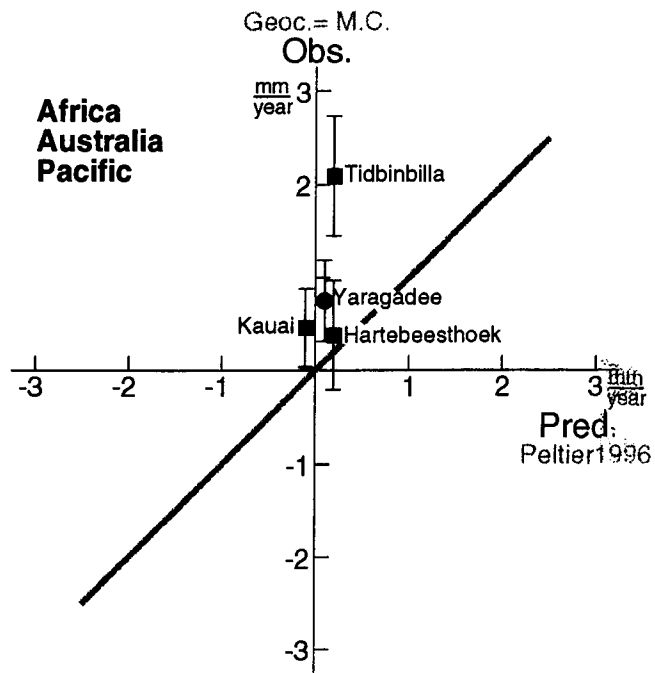
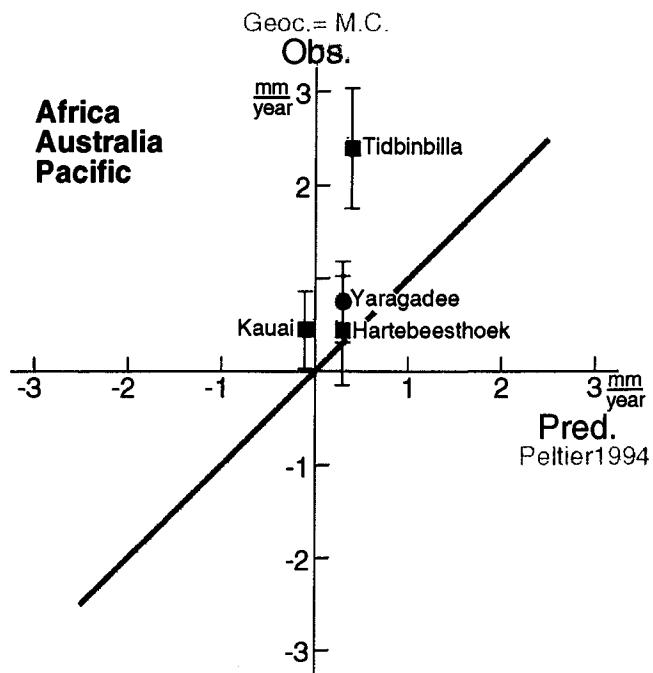
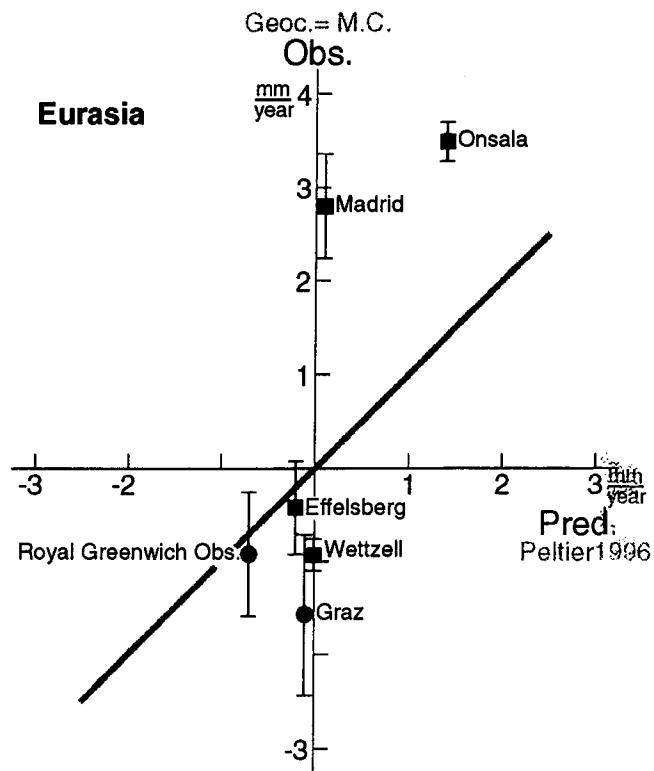
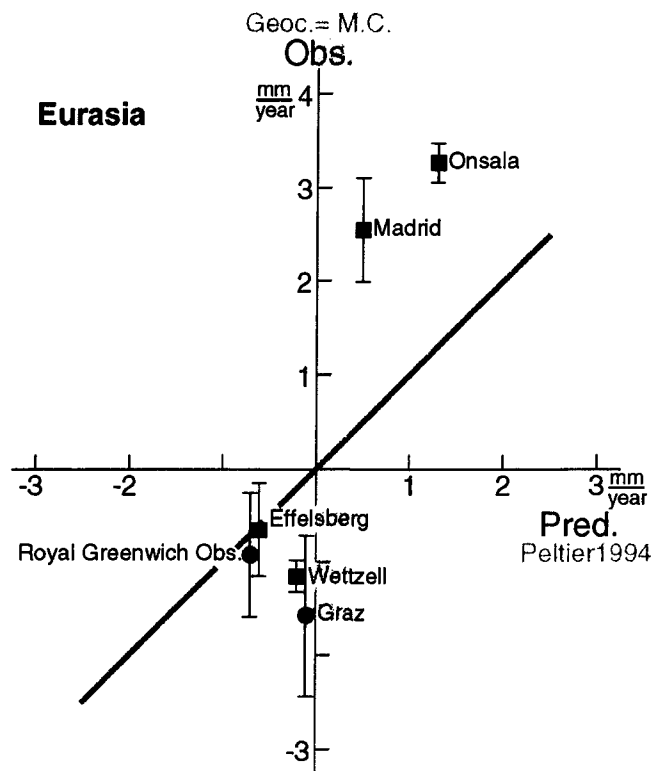


Figure 8 (cont.)

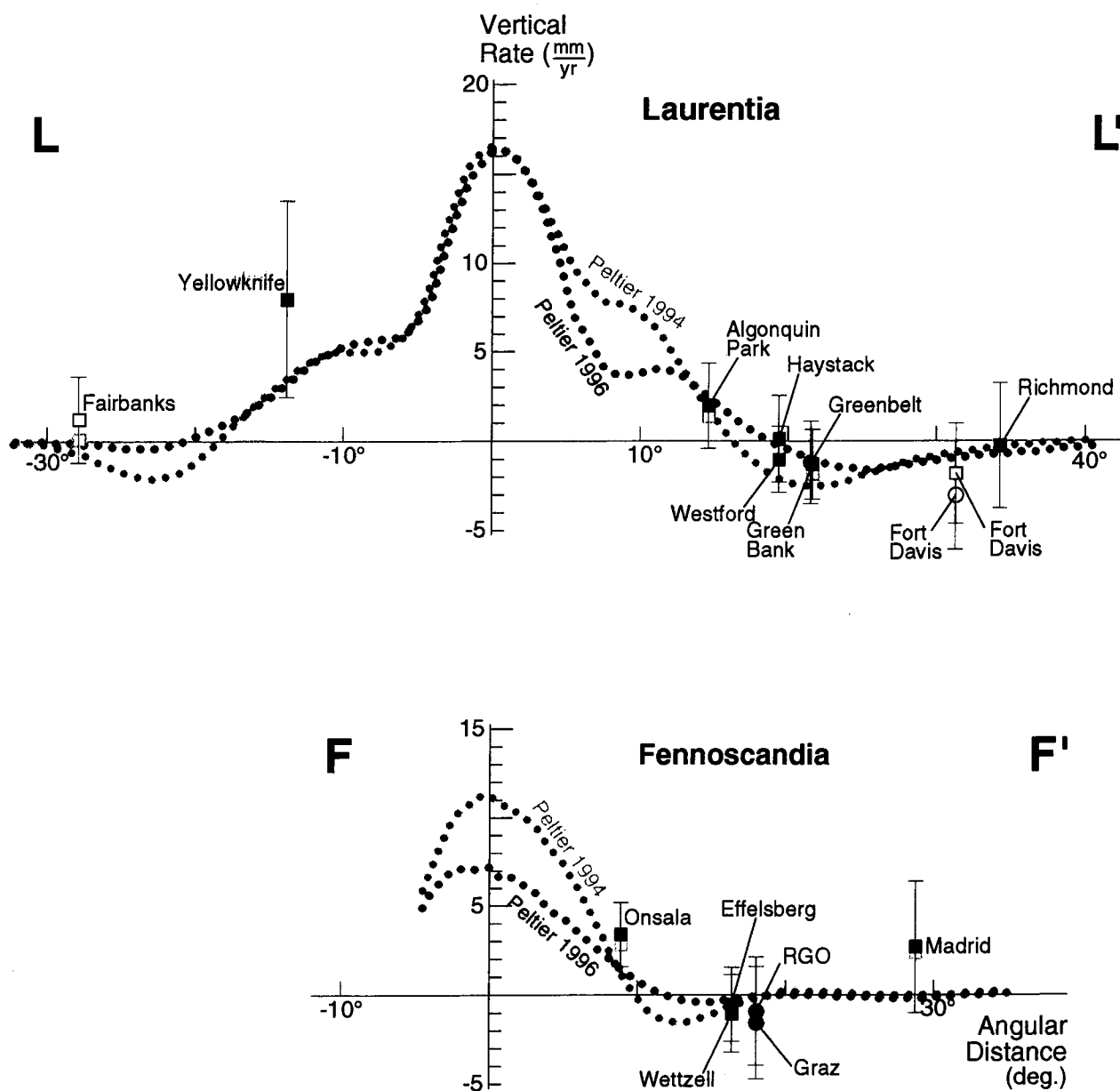


Figure 9

North America

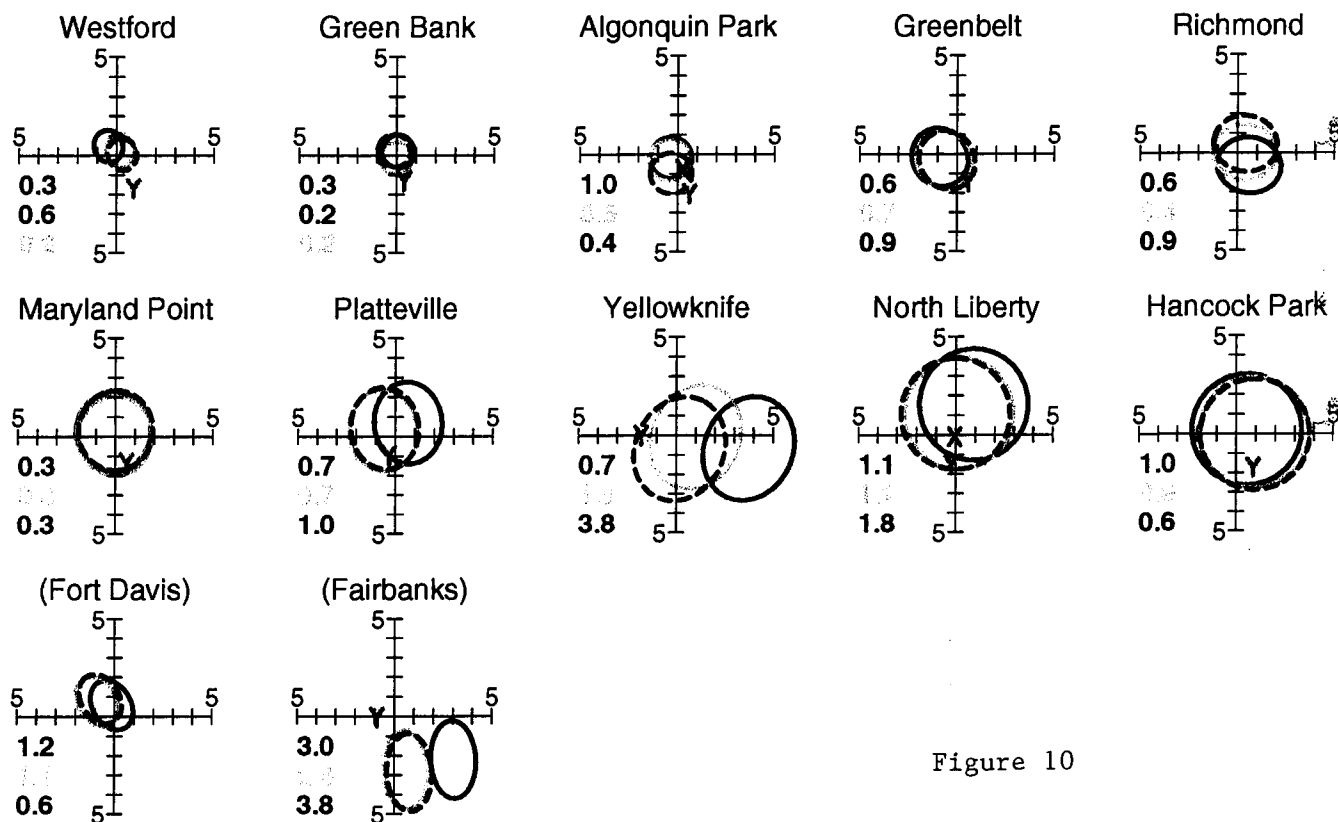
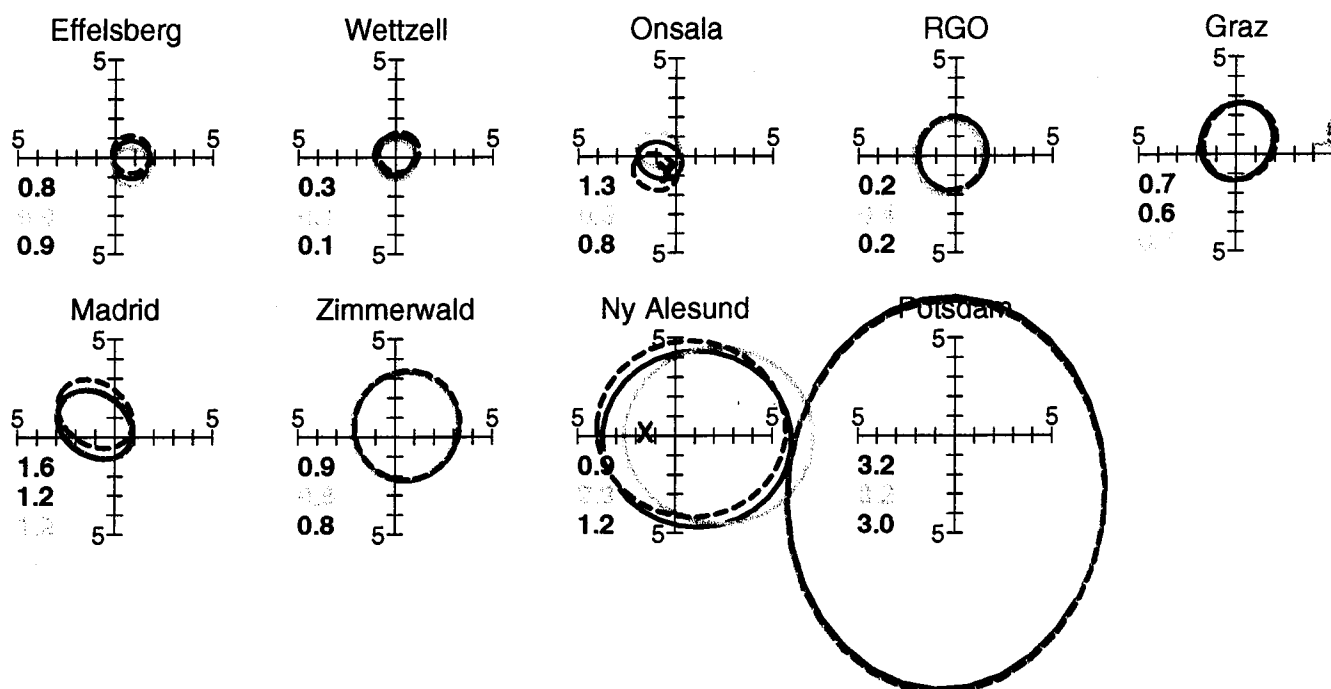


Figure 10

Eurasia



North America

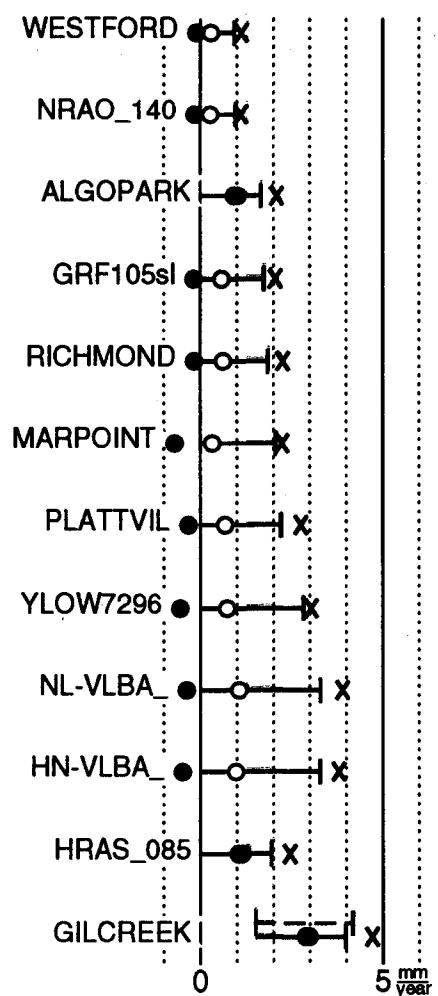
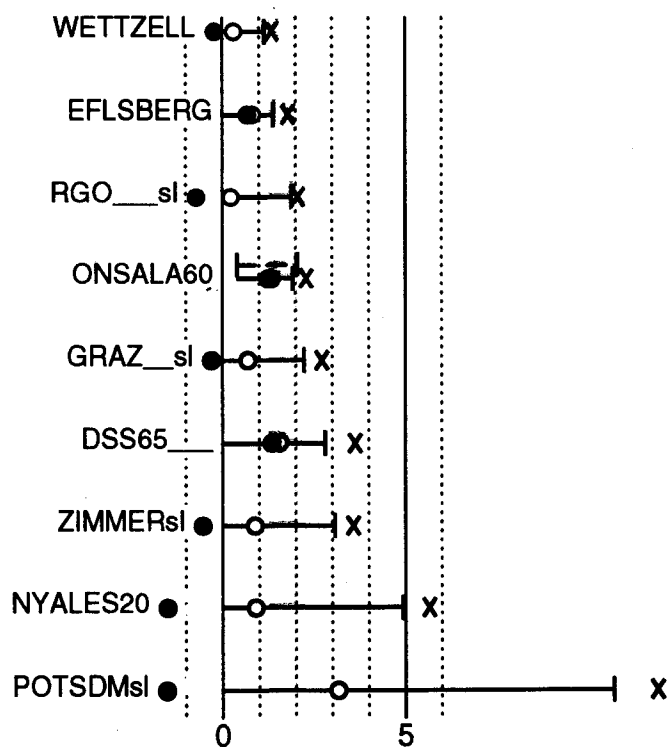


Figure 11

Eurasia



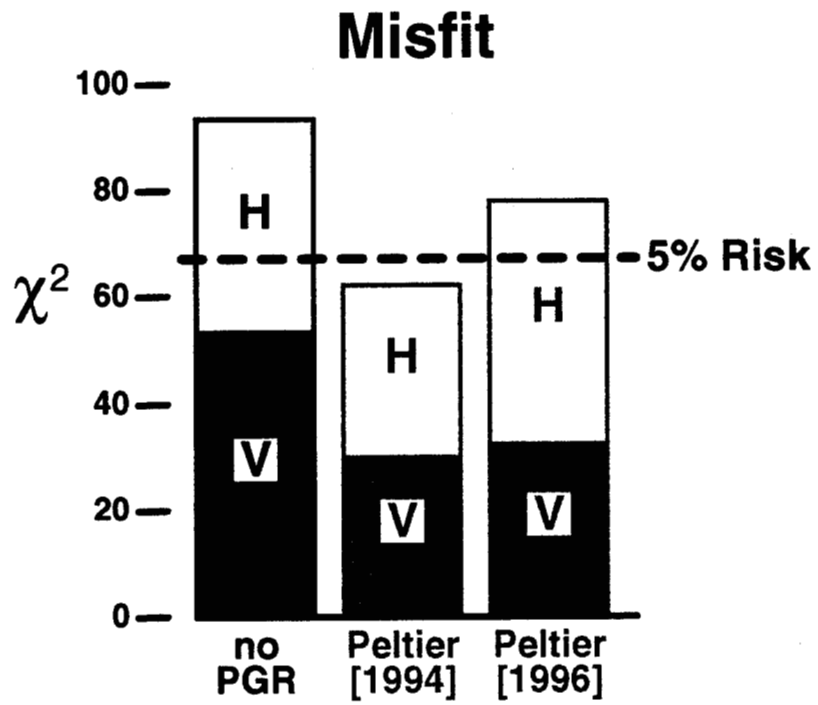


Figure 12

Table 1. Models implementing the various definitions of the reference frame.

| Model (nssd) | Data | Parameters Fixed | Parameters Estimated | Assumptions |
|--|---|---|---|--|
| Geoc. (P94 0.803, P96 0.909) | 29 VLBI site velocities (29x3), 14 SLR site velocities (14x3), less 3 anomalous SLR vertical rates (-3x1). 118 data. | angular velocity of North American plate | translational velocities of SLR and VLBI r.f.'s (2x3), angular velocities of SLR and VLBI r.f.'s (2x3), angular velocities of Australian, Eurasian, and Pacific plates (3x3), 2 components of the angular velocities of African, Antarctic, Nazca, and South American plates (4x2). 29 estimated parameters. | The uplift, subsidence, and deformation of the plate interi- ors is negligible after correcting for glacial isostatic adjustment. |
| C.M. (1.040) | 14 VLBI site velocities (14x3), 12 SLR site velocities (12x3), less 2 anomalous SLR vertical rates (-2x1). 76 data. | translational velocity of SLR r.f., angular velocity of SLR r.f. | translational velocity of VLBI r.f. (1x3), angular velocity of VLBI r.f. (1x3), veloc- ities of 12 sites (12x3). 42 estimated parameters. | VLBI and SLR sites at the same place have the same velocity. (At 10 places there is 1 VLBI and 1 SLR site. At 2 places there are 2 VLBI and 1 SLR site.) |
| Geoc.= C.M. (P94 0.817, P96 0.919) | 29 VLBI site velocities (29x3), 14 SLR site velocities (14x3), less 3 anomalous SLR vertical rates (-3x1). 126 data. | translational velocity of SLR r.f., angular velocity of North American plate. | translational velocity of VLBI r.f. (1x3), angular velocities of SLR and VLBI r.f.'s (2x3), angular velocities of Australian, Eurasian, and Pacific plates (3x3), 2 com- ponents of the angular velocities of African, Antarctic, Nazca, and South American plates (4x2). 26 estimated parameters. | The uplift, subsidence, and deformation of the plate interi- ors is negligible after correcting for glacial isostatic adjustment and the velocity of the geocenter equals that of the center of mass over decades. |
| Geoc.- NUVEL1A (1.808) | 29 VLBI site velocities (29x3), 14 SLR site velocities (14x3), less 3 anomalous SLR vertical rates (-3x1). 126 data. | angular velocities of North American, Eurasian, Australian, Pacific, African, Antarc- tic, South American, and Nazca plates. | translational velocities of SLR and VLBI r.f.'s (1x3), angular velocities of SLR and VLBI r.f.'s (2x3). 9 estimated parame- ters. | The velocities of the plates equal those in NUVEL1A and the uplift, subsidence, and deforma- tion of the plate interiors is neg- ligible. |

The normalized sample standard deviations (nssd's) describing the misfits of the models to data are listed in parenthesis in the left hand column. "P94" corrects for Peltier [1994]; "P96" corrects for Peltier [1996]. The normalized sample standard deviation is the square root of reduced chi-square (cf. equation in summary of chapter 10, Bevington [1969]).

The velocity of one site on a plate is not enough information to estimate the angular velocity of the plate. Therefore for the four plates with only one site we estimate two of the three components of the angular velocity of the plate. In these instances there is a 1 to 1 correspondence between the two horizontal components of velocity input and the two components of the angular velocity estimated; the horizontal data input are fit exactly and don't contribute toward the definition of the reference frame.

Table 2. Site Locations, Assignment of Sites to Plates, and Data Importances

| Site Abbrev. | Type | Lat. °N | Lon. °E | Hori. imp. | Vert. imp. | |
|-----------------------------|------|------------|------------|---------------|---------------|-------------------------------|
| North American plate | | | | | | |
| GRF105 | SLR | 39.02 | -76.83 | .77 | 0 | Greenbelt, Maryland |
| YLOW7296 | VLBI | 62.48 | -114.47 | .42 | .01 | Yellowknife, Northwest Terr. |
| WESTFORD | VLBI | 42.61 | -71.49 | .39 | .10 | Westford, Massachusetts |
| RICHMOND | VLBI | 25.61 | -80.38 | .37 | .03 | Richmond, Florida |
| PLATTVIL | SLR | 40.18 | -104.73 | .32 | 0 | Platteville, Colorado |
| NRAO 140 | VLBI | 38.44 | -79.84 | .30 | .08 | Green Bank, West Virginia |
| HAYSTACK | VLBI | 42.62 | -71.49 | .27 | .06 | Haystack, Massachusetts |
| ALGOPARK | VLBI | 45.96 | -78.07 | .26 | .06 | Algonquin Park, Ontario |
| PLATTVIL | VLBI | 40.18 | -104.73 | .28 | .01 | Platteville, Colorado |
| HAYSTK | SLR | 42.62 | -71.49 | .24 | omit | Haystack, Massachusetts |
| RICHMO | SLR | 25.61 | -80.38 | .22 | 0 | Richmond, Florida |
| MARPOINT | VLBI | 38.37 | -77.23 | .12 | .01 | Maryland Point, Maryland |
| NRAO85 3 | VLBI | 38.43 | -79.84 | .12 | .01 | Green Bank, West Virginia |
| NL-VLBA | VLBI | 41.77 | -91.57 | .12 | .01 | North Liberty, Iowa |
| HN-VLBA | VLBI | 42.93 | -71.99 | .10 | .01 | Hancock Park, New Hampshire |
| GGAO7108 | VLBI | 38.83 | -76.83 | .10 | .01 | Greenbelt, Maryland |
| NRAO20 | VLBI | 38.25 | -79.83 | .07 | .00 | Green Bank, West Virginia |
| | | | | 4.46 | .39 | |
| Eurasian plate | | | | | | |
| ONSALA60 | VLBI | 57.40 | 11.93 | .85 | .14 | Onsala, Sweden |
| DSS65 | VLBI | 40.43 | -4.25 | .63 | .03 | Madrid, Spain |
| WETTZELL | VLBI | 49.15 | 12.88 | .55 | .10 | Wettzell, Germany |
| EFLSBERG | VLBI | 50.52 | 6.88 | .53 | .10 | Effelsberg, Germany |
| RGO | SLR | 50.87 | .34 | .49 | 0 | Royal Greenwich Obs., England |
| GRAZ | SLR | 47.07 | 15.49 | .49 | 0 | Graz, Austria |
| ZIMMER | SLR | 46.88 | 7.47 | .28 | 0 | Zimmerwald, Switzerland |
| WETTZELL | SLR | 49.15 | 12.88 | .26 | 0 | Wettzell, Germany |
| NYALES20 | VLBI | 78.86 | 11.87 | .23 | .00 | Ny Alesund, Spitsbergen Isl. |
| POTSDM | SLR | 52.38 | 13.06 | .15 | 0 | Potsdam, Germany |
| | | | | 4.45 | .38 | |
| Australian plate | | | | | | |
| YARAG | SLR | -29.05 | 115.35 | 1.44 | 0 | Yaragadee, Western Australia |
| ORRLR | SLR | -35.64 | 148.94 | 1.07 | omit | Orroral, New South Wales |
| DSS45 | VLBI | -35.40 | 148.98 | .89 | .03 | Tidbinbilla, New South Wales |
| HOBART26 | VLBI | -42.80 | 147.44 | .84 | .02 | Hobart, Tasmania |
| PARKES | VLBI | -32.82 | 148.26 | .12 | .00 | Parkes, New South Wales |
| | | | | 4.37 | .05 | |
| Pacific plate | | | | | | |
| HOLLAS | SLR | 20.71 | -156.26 | 1.04 | omit | Maui |
| KAUAI | VLBI | 22.13 | -159.67 | .89 | .05 | Kauai |
| MARCUS | VLBI | 24.29 | 153.98 | .56 | .00 | Marcus Isl. |
| KWAJAL26 | VLBI | 9.40 | 167.48 | .44 | .00 | Kwajalein |
| HUAHI2 | SLR | -16.73 | -151.04 | .40 | 0 | Huahine |
| KOKEE | VLBI | 22.13 | -159.67 | .25 | .01 | Kauai |
| MK-VLBA | VLBI | 19.80 | -155.46 | .22 | .01 | Mauna Kea |
| | | | | 2.37 | .07 | |
| African plate | | | | | | |
| HARTRAO | VLBI | -25.89 | 27.69 | 2 | .06 | Hartebeesthoek, South Africa |
| Antarctic plate | | | | | | |
| OHIGGINS | VLBI | -63.32 | -57.90 | 2 | .01 | O'Higgins, Antarctica |
| South American plate | | | | | | |
| FORTLEZA | VLBI | -3.88 | -38.43 | 2 | .01 | Fortaleza, Brazil |
| Nazca plate | | | | | | |
| EASTR2 | SLR | -27.15 | -109.38 | 2 | 0 | Easter Isl. |
| | | | | 25.02 | .98 | Total |
| | | | | 15.88 | .98 | VLBI Subtotal |
| | | | | 9.14 | 0 | SLR Subtotal |

'Vert. imp.' is the importance of the vertical rate of a site and 'Hori. imp.' is the importance of the horizontal velocity of a site. Data importances are computed using Minster et al. [Equation 19, 1974]. The importance of a datum is a measure of what fraction of a parameter the datum is constraining. The data importances listed are for the model assuming that the uplift, subsidence, and deformation of the plate interiors is negligible after correcting for rebound and that the velocity of the geocenter relative to the center of mass is negligible when averaged over decades (model "Geoc.= C.M."). The SLR translational velocity is fixed; therefore the SLR vertical data have zero importance. The VLBI translational velocity is constrained by the VLBI vertical (imp.= .98, 33%) and VLBI horizontal (imp.= 2.02, 67%) data. The angular velocities between the plates are constrained by the VLBI horizontal (imp.= 10.86, 63.9%) and the SLR horizontal (imp.= 6.14, 36.1%) data.

Table 3a. Formulae for additional systematic error

| | Horizontal | Vertical |
|------|--|--|
| VLBI | $\frac{5 \text{ mm}}{\text{time yr}}$ | $\frac{15 \text{ mm}}{\text{time yr}}$ |
| SLR | $\frac{2 \text{ mm}}{\sqrt{\text{time yr}}}$ | $\frac{5 \text{ mm}}{\sqrt{\text{time yr}}}$ |

We formulate a realistic error budget by incorporating an additional systematic error computed using different formulae for the different data types. For example, the additional systematic error for a VLBI horizontal datum is equal to a distance of 5 mm divided by the time period of observations at the site in years. The systematic error in a component of horizontal velocity for a VLBI site with 10 years of data would be .5 mm/yr (= 5 mm/10 yr). The square of the systematic error is added to the square of the formal error to obtain the revised variance. See Argus and Gordon [1996].

Table 3b. Normalized sample standard deviations describing misfits of the model of Peltier [1994] to different groups of data

| | Horizontal | Vertical |
|------|------------|----------|
| VLBI | 0.801 | 0.896 |
| SLR | 0.919 | 0.724 |

The normalized sample standard deviation is the square root of reduced chi-square and is computed using the usual formula except that the total importance of the group of data is substituted for the number of parameters. The nssd's for all four groups of data is slightly less than one, indicating that the errors are conservative.